Virtual Brands and Platform Intermediation*

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Abstract

Virtual brands, established by firms to sell their existing products on online platforms, are gaining prominence on various food-delivery platforms. This paper studies a firm's decision to create multiple brands on an online platform and the platform's decision to recommend them to consumers who make their purchase decisions after searching recommended brands on the platform. We find a multi-product firm can utilize multiple identical-menu brands with different leading products to communicate information about its product variety, enticing more consumers to search its brands on the platform. Surprisingly, such information transmission by a multi-product firm raises not only consumer surplus but also the profit of the single-product firm that does not utilize virtual brands. We find that under privacy environment where the platform does not have access to consumer-preference information, it facilitates this information transmission by consistently recommending all brands to all consumers. By contrast, under no privacy where the platform knows consumer types, it uses brands with different leading products to target different consumer segments, which essentially restricts the information-transmission channel. Interestingly, however, the profits of both the platform and the virtual-brand-offering firm increase as a result. Finally, we show banning identical-menu virtual brands can further benefit both the multi-product firm and consumers when the ban pushes the multi-product firm to create multiple virtual brands specializing in distinct products. However, when the ban leads the multi-product firm to keep only one brand with all its products, the ban can hurt consumers and all firm types.

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1 Introduction

The online food delivery industry is growing rapidly, with global revenues reaching $436.6 billion in 2024 (Statista, 2025). Major platforms such as UberEats, Grubhub, and DoorDash dominate the market, facilitating transactions between consumers and restaurants (McKinsey, 2021). In recent years, these platforms have promoted the creation of "branded virtual restaurants" or "virtual brands" for selling food on the platform (Grubhub, 2025). Denny's introduced The Melt Down and Burger Den, Chili's launched It's Just Wings, and Hooters established multiple virtual brands, including Hootie's Burger Bar, Hootie's Bait & Tackle, and Hootie's Chicken Tenders (CNBC, 2021). The number of virtual brands on UberEats alone quadrupled from 10,000 in 2021 to over 40,000 in 2023 (Restaurant Dive, 2023). This trend extends beyond large chains—Figure 1 shows two virtual brands (on UberEats) offered by the same physical location of a local Toronto restaurant chain.

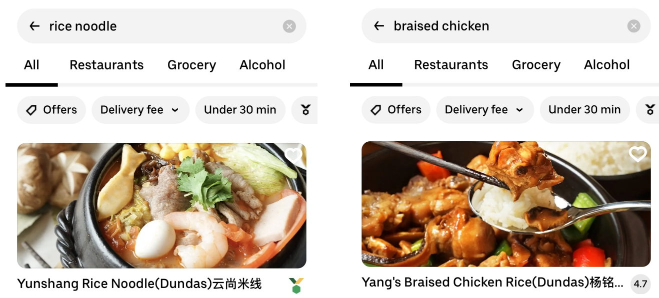

Figure 1: Virtual brands "Yunshang Rice Noodle" and "Yang's Braised Chicken Rice" on UberEats offered by a Dundas Steet restaurant in Toronto

Unlike the traditional "umbrella" branding strategy, where firms introduce new products or target new consumer segments under a single brand, as pointed out in Wall Street Journal (2023), firms may create multiple virtual brands to "draw customers' attention to their many offerings. An Indian restaurant . . . might create multiple brands with the same menu but give each a different name and lead photo."

However, since 2022, platforms have begun restricting virtual brands with identical menus, requiring a minimum percentage of unique items per brand from the same physical location. For example, DoorDash mandates at least 50% menu differentiation, and Grubhub follows similar guidelines (Business Insider, 2022).[1] UberEats removed about 8,000 virtual brands that failed to differentiate their menus, while others adjusted to meet the new guidelines (Business Insider, 2023). Among the virtual brands removed by UberEats, there were 12 brands selling identical breakfast burritos from a Colorado sports bar and 14 brands serving the same sandwiches from a New York City deli (Wall Street Journal, 2023). Interestingly, as of March 2025, both of the virtual brands shown in Figure 1 continue to offer identical menus with different leading products.

[1] See DoorDash requirements: https://help.doordash.com/merchants/s/article/Official-DoorDash-Virtual-Brand-Quality-Requirements.

What drives firms to create seemingly duplicate brands, and why do platforms both promote and restrict them? In platform-driven markets, recommendation algorithms determine which brands consumers see, influencing search results and visibility. If platforms are concerned about consumer confusion or some firms flooding the market with a lot of brands, they could simply filter excess listings rather than banning them outright. Likewise, if virtual brands serve to target different consumer segments, recommendation algorithms with exploration mechanisms could achieve the same goal without requiring multiple brands.

To investigate this novel marketing phenomenon and shed light on the role of multiple brands on online platforms, why a platform may regulate identical-menu virtual brands and how these choices affect all relevant stakeholders, we examine the interaction between the firm's creation of multiple brands and the platform's selective recommendation. The objectives of this paper are threefold: first, to find the benefits that firms accrue from creating multiple brands; second, to analyze the impacts of the multiple brands on consumers, firms that do not create multiple brands, and the platforms; and third, to explore the potential consequences of the platform allowing or banning virtual brands that use menus identical to the original brands.

We develop a model in which consumers buy products from a firm through a platform. There are two types of products and two types of consumers, each familiar with one product type and unfamiliar with the other. Familiarity is defined based on whether a consumer knows her utility for a product ex ante. A firm may be capable of producing only one or both of these two products and need to create brands for selling them. In this paper, we focus on the online market mediated through the platform, so we do not distinguish virtual brands and the original brand. Instead, the technology of virtual brands simply make the creation of multiple brands possible and costless for the firm. The firm can decide on the number of brands to create, and for each brand, select a leading product that is visible to consumers prior to their search. Consumers conduct searches at the brand level, incurring a search cost for each brand. The platform, which collects a percentage fee proportional to firm revenue, assesses the consumers' preference types and presents a subset of the available brands to each consumer. The recommendation can be either type dependent or independent. Before initiating a search for any brand, consumers only observe the brands recommended to them, as well as the leading product of each brand. Upon paying the search cost to visit a brand, they are able to observe the product variety, the prices of products within the brand, and the exact utility of the unfamiliar product, if it is offered by the brand.

We find a novel informational channel of product variety when there are multiple products. In equilibrium the two-product firm launches two distinct brands, each offering both products but featuring a different leading product. By contrast, the single-product firm creates just one brand for its only product. Even though a two-product firm prices the same product identically across both brands, and consumers limit their search to a single brand, the mere existence of a second brand, with a different leading product, serves to communicate information on product variety, distinguishing the firm from its single-product counterparts.[2] Such separation boosts the expected

[2]One could argue that the firm might communicate product variety by using different product images as the

likelihood of consumers finding a desirable product, making them more inclined to visit such a brand. In the meantime, the firm can raise prices for both products, caring less about which specific product to sell. The added brand, seemingly redundant, actually carries informational value.

The platform plays a pivotal role in this information communication by recommending all brands to all consumers. In the privacy environment where the platform does not know or cannot make use of consumer-preference information to target consumers, the platform finds sustaining this informational channel is beneficial. It mitigates information asymmetry between consumers and the firm by displaying all available brands, enabling the firm to use virtual brands to communicate product variety. Surprisingly, our findings indicate the creation of multiple brands by the two-product firm contributes to an overall improvement in welfare, benefiting also consumers and the single-product firm. Consumers benefit by making informed search decisions, intensifying their search efforts when the expected surplus is high and reducing them when the surplus is lower. The benefits accruing to the single-product firm as a result of its separation from the two-product firm are more nuanced. As a seller of only one product, the single-product firm is compelled to set a lower price to cater to the demand from both consumer types. As a result of not pooling with the two-product firm, the single-product firm faces disproportionally lower demand from consumers familiar with its product and would hence lower its price. The lower expected price instead leads to increased consumer visits, proving advantageous for the firm. The platform would, by extension, benefit from this "win-win-win" equilibrium.

In comparison, in the no privacy environment where the platform has consumer-preference information and can use it to target consumers, the platform further benefits from making targeted recommendations. This finding highlights the second function of virtual brands: the multiple brands created by the two-product firm with distinct leading products enable the platform to target consumers. The platform would show only one of the brands to each consumer depending on their types. In this equilibrium, the firm loses its ability to use virtual brands to transmit product-variety information to consumers. Interestingly, the profits of both the platform and the two-product firm increase as a result. The targeted recommendation aided by the firm's creation of distinct brands makes the beliefs of heterogeneous consumers diverge prior to their search. Through targeted recommendation, the platform effectively guides consumers to intensify their search when the firm is better at making profits, that is, when the firm offers multiple products. The direction of consumer traffic and change in consumer composition benefits the virtual-brand-offering firm even without change in its product prices.

Our policy implication is that the ban of identical-menu virtual brands can elevate welfare for both the firm and consumers. In scenarios where the utility of the familiar product is moderate, a two-product firm would establish two separate brands, each offering a different product. Consumers

leading image or by including all product names in the brand name. However, due to limited screen space and other practical constraints, this approach is often not desirable. However, we do see firms try to convey variety information in other ways when it is easy to do so.

self-select to visit the brand offering their unfamiliar product first and may opt to explore the other brand. The creation of two distinct brands, each offering just one product, curtails the two-product firm's ability to exploit consumers, forcing the firm to lower prices as consumers self-select to visit the brand selling their unfamiliar product first. The two-product firm has incentive to further reduce price to persuade consumers continue search the second brand if the first one is not a good match. Interestingly, expectation of lower prices serves to attract more consumers to visit, resulting in a welfare improvement for both the two-product firm and consumers. However, achieving such a welfare-enhancing outcome necessitates the ban by the platform, because the two-product firm itself is otherwise inclined to offer both products in its brand and raise prices to maximize surplus extraction from consumers, which forms a hold-up problem. The ban effectively provides a commitment device that limits the firm's ability to exploit consumers conditional on search, which in turn attracts more consumer traffic. Note that this equilibrium featuring two distinct brands is only feasible when the utility of the familiar product is not very high, ensuring consumers are indeed inclined to initiate their search with the brand selling their unfamiliar product. Otherwise, the equilibrium scenario depicts the two-product firm opting to create a single brand offering both products, which is a no-virtual-brand equilibrium.

The information channel of product variety is also prevalent in other online marketplaces. On Amazon, some sellers create multiple product listings for the same set of products, varying the lead image by color or design (Figure 2). Additionally, Amazon now provides tools for sellers to directly display some easy-to-show variety information, such as color options, on the search results page. These multiple listings with varying leading images, similar to virtual brands in food delivery, help firms communicate product variety to consumers.[3] This variety information influences consumer search behavior, shaping firms' branding and pricing strategies. Thus, insights from virtual brands provide a broader perspective on how product variety information affects firms and consumer engagement across digital marketplaces.

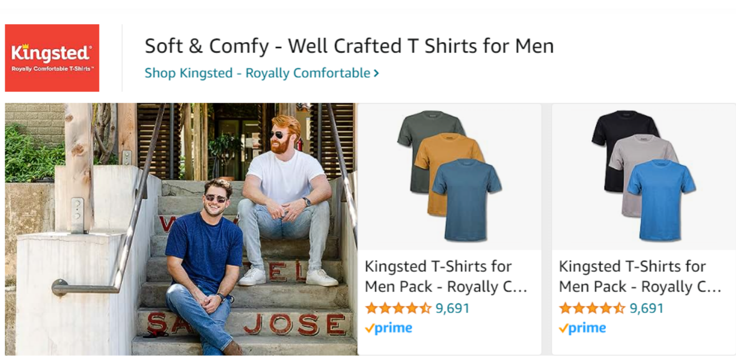

Figure 2: Multiple product listings of the same group of products on Amazon

[3]While the price of the lead product is sometimes displayed, the prices of other items within the same group—often varying—are not immediately visible, requiring consumers to search for them.

1.1 Literature review

Our paper explores the novel marketing phenomenon of virtual brands and naturally contributes to the branding literature in marketing and economics. Traditionally, a brand is seen as a firm's identity tied to its products, and the literature often views branding as a way to convey vertical product information either through signaling or reputation concerns when consumers have incomplete information about product quality prior to consumption (Bronnenberg, Dubé, and Moorthy, 2019). Branding and brand advertising can serve as signals for product quality when there is a correlation between quality and advertising spending (Nelson, 1970; Kihlstrom and Riordan, 1984; Milgrom and Roberts, 1986). From a dynamic perspective, brands give high-quality-product manufacturers incentives to uphold their reputation through consistent branding (Klein and Leffler, 1981; Shapiro, 1983; Tadelis, 1999). The signaling of product quality—whether through pricing, advertising, or other mechanisms—has also been extensively studied in the marketing literature (Simester, 1995; Miklós-Thal and Zhang, 2013; Guo and Wu, 2016; Chen and Liu, 2022; Kraft and Rao, 2025). Our paper contributes to this literature by uncovering a novel dimension of informational value: rather than signaling product quality, branding can serve as a mechanism for signaling product variety, influencing consumer search and choice. We show how multi-product firms can leverage virtual brands to disseminate information about their product variety, influencing consumer search behavior and the pricing strategies of all types of firms. Also on signaling about vertical information, a large body of literature focuses on umbrella branding and brand extension as means of conveying quality information when introducing a new product to a different market (Wernerfelt, 1988; Wernerfelt, 1991; Cabral, 2000; Moorthy, 2012; Yu, 2021). In both models on umbrella branding and our model, the ownership of the brands or the identity of firms are observable to consumers, which guarantees the effective communication of information to consumers. Our study, however, focuses on the same product market, and the two products are substitutes for consumers with unit demand in this market. Instead of quality information, our focus is on variety information transmitted through additional brands.

A few recent papers also study branding from various perspectives, for example, the impact of social factors such as match making and status goods in branding decisions (Kuksov, 2007; Amaldoss and Jain, 2015), strategic decisions of store brands versus national brands (Sayman, Hoch, and Raju, 2002; Scott Morton and Zettelmeyer, 2004; Soberman and Parker, 2006; Amaldoss and Shin, 2015), and brand positioning in horizontal differentiation models (Ke, Shin, and Yu, 2023). In an era where digital feedback and reviews are plentiful, thus eroding the traditional signaling value about quality (Bronnenberg, Dubé, and Moorthy, 2019), our paper posits that branding may carry informational value through another channel, such as variety information, that also impacts firms' pricing strategies. Several recent papers also analyze the activity of firms creating multiple brands from different angles and nicely complement our paper (Armstrong and Vickers, 2024; Grubb and Westphal, 2024). Their analyses focus on a single product, and consumers may not be aware of multiple brands selling the same product either through different exogenous consideration sets or through different search probabilities. They do not consider the role of the platform. By contrast, our paper highlights the effect of multiple products and the platform's selective recommendation by the platform. No consumer confusion exists about the brand ownership, and the awareness of brands depends on the endogenous decisions of the firm and the platform. In the online marketplace, if the concern is consumer confusion or market congestion, platforms could arguably manage brand visibility through their recommendation systems, potentially eliminating the need for additional branding restrictions.

Our research is related to the expansive literature on consumer search, starting from the seminal paper Diamond (1971). Wolinsky (1986) introduces sequential search with horizontal differentiation. In our model, consumers are heterogeneous in the product familiarity and utility of the unfamiliar products. They search sequentially among different brands created by a firm and filtered and presented by a platform. In Varian (1980), firms also face both informed and uninformed consumers, similar to our notion of familiar and unfamiliar product. However, their notion of familiarity is whether consumers know the price before search and the utility is fixed, whereas our notion is whether consumers know the utility of the product. Arbatskaya (2007) and Armstrong, Vickers, and Zhou (2009) study directed search. In the former, the entire search sequence is exogenously set, whereas in the latter, a prominent firm is exogenously selected to be searched first and the rest of the search sequence is random. By contrast, in our model, brands and their leading products are visible to consumers and consumers direct their search first to the brand that gives them highest expected consumer surplus. Mayzlin and Shin (2011) and Yao (2024b) show that firms strategically decide whether to advertise a particular attribute to influence consumer search behavior, encouraging consumers to explore other attributes of the same product. Yao (2024a) takes an information design approach, directly examining how firms design information structure to shape consumers' search decisions. By contrast, our framework finds the two-product firm can communicate product variety information to attract more consumers to search.

Our paper also contributes to the recent literature on search design, information transmission, and recommendation strategies within the platform economy. Hagiu and Jullien (2011) explores a scenario where a platform controls the search sequence for heterogeneous consumers between two single-product firms. By contrast, our paper positions the platform to selectively recommend a list of brands, whereas consumers retain their control over their search sequence among these brands. Dukes and Liu (2016) considers the design of the online marketplace hosting multiple sellers with ex-ante homogeneous product, and the platform can directly alter the search cost for consumers. In our paper, the search cost is exogenous to the platform and the design is the recommendation list. Jiang and Zou (2020) delve into a platform's optimal setting of transaction fees and assess the impacts of lowered search costs and filtering based on observable attributes. In Ke, Lin, and Lu (2023), the platform directly uses information design to change consumers' belief about their matched product before they search and firms bid to win a prominent position. Zhong (2023) investigates a threshold model for product filtering based on match value and price, examining how a platform balances these factors depending on precision. In comparison to these papers, in our model, the percentage fee is fixed and our research is centered on selective recommendations by the platform and restrictions imposed on virtual brands. Additionally, Zhou and Zou (2023) examine how personalized product recommendations by online marketplaces influence seller competition, considering different levels of consumer profiling accuracy. Their model features a platform recommending a single product to consumers, who may or may not be aware of other available products outside the recommendation. Long and Liu (2024) explores how platforms manipulate product attractiveness and organic ranking, where a certain percentage of consumers follow the platform-designed ranking order when choosing among competing sellers. Our study differs by analyzing a setting where the platform recommends a list of brands, making both the brands and their leading products directly observable to consumers. However, discovering prices and product variety within each brand requires search effort, which incurs a cost. Unlike models where the platform dictates ranking order, here, consumers freely determine their search order and consider all products provided in searched brands.

2 Model

Consider a firm that sells an uncertain variety of products in a market consisting of a unit mass of consumers. For simplicity, suppose there are only two possible products, $i \in \{1, 2\}$. The firm may be capable of producing one or both products. With probability γ, the firm can produce only one of the two products (either product 1 or 2). We refer to such a firm as a single-product firm. If a single-product firm can produce only product 1, we refer to it as a j_1 firm, labeled as $t = j_1$. Similarly, a firm that can produce only product 2 is referred to as a j_2 firm and is labeled as $t = j_2$. A single-product firm can be a j_1 firm or a j_2 firm with equal probability. With probability $1 - \gamma$, the firm can produce both products, and we refer to such a firm as a two-product firm, labeled as $t = j_{12}$. The firm's product variety is its private information. The marginal cost of production is zero for both the products.

The firm sells products to consumers using an independent online platform. When a consumer arrives at the platform to buy a product, the platform presents a list of brands to her. Each brand consists of a menu of products and their prices. However, the list of brands presented to the consumer displays only a product image (which is referred to as the image of the leading product) for each brand in the list. The information about all the products offered within the brand and their prices become known to the consumer only when she searches that specific brand by clicking at it. A key feature of our model is that consumers search sequentially at the brand level (and not at the firm or product level). To reach potential consumers, the firm establishes brands on the platform. When virtual brands are not available, each firm can just create one brand; when virtual brands are available, each firm can create multiple brands. The firm decides the number of brands, the product menu offered within each brand (subject to its own product variety), the price for each product within each brand, and the leading product for each brand.[4] We represent the price of a single-product leading-i brand for its sole product i by p_{s_i} and prices for a two-product brand by a pair (p_1, p_2), where p_i is the price of product i. We assume the cost of introducing a new brand is

[4] The main model focuses on a monopoly firm. In Section 5.1, we consider how virtual brands affect equilibrium with competitive firms.

infinitesimally small, implying the firm will establish a brand only if doing so yields strictly higher profits.

Consumers are potentially interested in these two products and have unit demand, but they vary in their preferences between the two. Specifically, there are two consumer types, $\theta \in \{L, R\}$, present in equal proportions in the population. A type-L consumer is familiar only with product 1, whereas a type-R consumer is familiar only with product 2.[5] A consumer's valuation v for her familiar product is deterministic. However, a consumer's valuation u for her unfamiliar product is stochastic, and $u \sim U[0, 1]$ is independent across consumers. We assume $v > 1/2$. That is, consumers have a higher expected value for their familiar product than their unfamiliar product. We consider the opposite case where the familiar product offers a lower expected value, $v < 1/2$, in Section 5.2. A consumer's product valuations are specific to products and do not depend on the brands they are offered in. Although v is publicly known by all players ex ante, the realization of u is learned only privately by each consumer after they search a brand containing their unfamiliar product. A consumer buys at most one unit of the product.

We consider two information environments: a privacy environment where each consumer's preference type θ is privately observed by the consumer herself, and a no privacy environment where each consumer's preference type θ is observed by both the consumer and the platform. The platform's information on consumer type may come from consumers' keyword search or behavioral tracking of past activities. The platform displays a non-empty subset of available brands to each consumer. Consumers learn about only those brands the platform displays to them. Consumers observe the leading product i of each displayed brand and form a belief β^i about the brand being a single-product brand. Note β^i is an endogenous variable and is derived using Bayes' rule wherever possible.

Subsequently, a consumer can incur a search cost $c \sim U[0, \bar{c}]$ (where $\bar{c} \geq 1$) per brand to learn the menu of product/s (and their price/s) offered within that brand.[6] Consumers try to find their best purchase option by searching sequentially. The search cost c is individually and independently drawn and independent of consumer preference type θ.[7] The search cost c is privately observed by each consumer before search. Upon searching a brand, consumers discern the exact utility of each product offered in that brand. Each consumer decides whether and which brand to search first, whether to search a second brand, and whether to buy a product or leave the market without any purchase after each search. The number of brands and the leading product of each brand are observable to consumers before they search. Therefore, they may affect consumers' sequential search decision. The product variety—other than the leading product—and product prices are

[5] Introducing some degree of unfamiliarity helps to circumvent the Diamond paradox. The Diamond paradox says that when consumers share homogeneous valuations and have positive search costs, the unique equilibrium of the sequential search is that the firm sets the monopoly price and consumers do not search at all.

[6] The assumption $\bar{c} \geq 1$ ensures some consumers who decide not to search exist in the market. This assumption also simplifies the analysis.

[7] We model the privacy environment based solely on preference type θ rather than search cost c. This focus is due, first, to the greater importance of preference type for the firm, and second, to the fact that preference information can be more easily learned over time, whereas search cost is more variable.

unobserved to consumers before they search. Therefore, they can affect consumers' decisions only through consumers' expectations.

The platform levies a transaction fee on the firm (for using the platform to sell its products) that extracts a proportion of the firm's revenue (or, equivalently, its profit under the current model setup). Therefore, from the platform's perspective, maximizing its own profit is equivalent to maximizing the sum of expected profits of all firm types.[8] The platform decides which brands to recommend (or display when they arrive at the platform) to consumers. The platform can offer either a non-targeted or targeted recommendation. A recommendation is non-targeted if the platform displays the same subset of brands regardless of the consumer's familiar product. In the case of a targeted recommendation, the platform recommends different subsets of brands to different types of consumers. Note that in our model, we assume the platform only presents an unordered subset of brands to each consumer so that the order of brands within the subset does not convey any information.[9] The search order within a list of brands is decided entirely by consumers instead of the platform, which is different from the literature on prominent search (Armstrong, Vickers, and Zhou, 2009; Hagiu and Jullien, 2011).

The platform's recommendation strategy falls into one of three categories: (i) recommending all available brands to all consumers (full recommendation), (ii) recommending the same subset of brands to all consumers (non-targeted selective recommendation), or (iii) recommending different subsets of brands to consumers based on their familiarity with specific products (targeted recommendation). Only the first strategy reveals all available brands to consumers; the other two strategies filter brand information, for example, by showing only one brand when multiple brands exist on the platform. While the second category may seem redundant if the firm does not create multiple brands, including it ensures a complete consideration of the platform's strategy set and highlights the potential value of the platform's ability to commit to a specific selection of brands. The firm, however, cannot commit to a predetermined probability mix for creating different brands. The platform selects the recommendation strategy that maximizes its expected profit. In a privacy environment, the platform can choose only from non-targeted recommendation strategies, whereas in a no-privacy environment, all three strategy categories are available.

The timing of decisions is as follows. First, the platform commits to a strategy for recommending brands to consumers.[10] Next, the firm decides the number of brands, the product variety and the leading product for each brand, and the prices of all products within each brand. The platform then presents a list of brands to consumers according to its recommendation strategy. Consumers observe the list of brands recommended to them on the platform and form expectations about

[8]In the model presented in Section 5.3, we study an alternative setup where the platform receives a fixed per-transaction fee. As a result, the platform's objective in that setup is to maximize total transaction volume.

[9]If the platform can present a ordered list, it can also target consumers with the same subset but different orders of brands. As it turns out, with consumers ultimately deciding the search order, targeting using the same subset but different orders of brands is not effective and does not change the equilibrium outcome. Therefore, for ease of exposition, we assume the subset of brands is unordered.

[10]Because the platform and the two-product firm often share a common interest, the platform's ability to commit to a recommendation strategy and its effect on the creation of multiple brands is straightforward.

product variety and product prices. Subsequently, consumers decide the order of search. Then, they sequentially search the list of brands until deciding to buy a product or opting out. Finally, payoffs are realized.

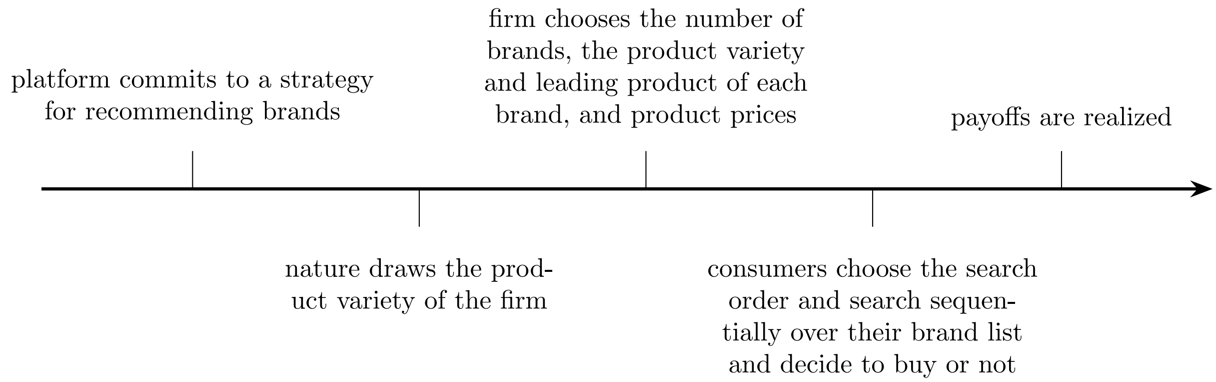

Figure 3: Timing of the game

We look for the extensive-form Trembling Hand Perfect Equilibrium (THPE) of the game (Selten, 1975) and adopt the intuitive criterion (Cho and Kreps, 1987). The equilibrium consists of a specification of the platform's recommendation strategy, the firm's branding and pricing strategies, consumers' search strategies, and their beliefs about product variety and product prices for each brand before they search. Consumers would form an expectation of the product variety offered by each brand, based on the brands recommended to them and their leading products, which is the result of both the firm and platform's strategy, and form an expectation of the product prices. When consumers are indifferent among several options, we assume they will choose each of these options with equal probability. Here, the brands and their leading products may serve as an information-communication device for the firm. THPE requires that every move at every information set is taken with nonzero probability. Consequently, each brand introduced by the firm may be visited by either type of consumer with nonzero probability, ensuring that the firm optimally selects product variety and product prices for each brand. The intuitive criterion states that if consumers were to observe an off-equilibrium action from any firm type, they would only ascribe it to firm types that may benefit from the deviation.[11]

3 Equilibrium Analysis and Results

In this section, we present the equilibrium analysis and generate insights about the firm's and the platform's decisions. We start with the analysis of consumer and firm strategies that can possibly be chosen in an equilibrium. Subsequently, in Section 3.2, we assume the platform recommends all

[11] Consumers observe the number of brands and the leading product of each brand, leading to a wide range of possible off-path strategies for the firm in any equilibrium. For example, consider a candidate equilibrium where the j_1 firm creates two leading-1 brands, the j_2 firm creates a leading-2 brand, and the j_{12} firm creates both a leading-1 brand and a leading-2 brand. An off-path strategy could involve a firm creating three leading-1 brands. In such cases, consumers must infer whether the firm following this off-path strategy is a j_1 firm or a j_{12} firm. The intuitive criterion helps refine the set of equilibria we consider by ruling out implausible consumer beliefs and firm deviations.

available brands to consumers and solve the candidate equilibrium in which the two-product firm creates two brands. Section 3.3 analyzes the candidate equilibrium in which the two-product firm creates only one brand. Section 3.4 presents insights about the effects of creating virtual brands. Finally, Section 3.5 presents the analysis and insights about the platform's strategies under privacy (that is, when the platform cannot use consumer preference information) and no-privacy settings.

3.1 Consumer and firm strategies

A consumer is presented with a list of brands and observes each brand's leading product. Based on this limited information, she forms expectations about product variety and prices, calculating her expected surplus from visiting each brand. She will first visit the brand with the highest expected surplus, provided her search cost is lower than the expected surplus. Upon visiting, she may either purchase a product from this brand, leave the market without buying, or pay an additional search cost, c, to visit another brand, if available.

For instance, consider the scenario in which a type-L consumer only sees one leading-1 brand, holds belief β^1 of it providing just product 1, and expects the prices to be p_{s_1} for a single-product brand and (p_1, p_2) for a two-product brand where p_i is the price of product i and $v - p_1 + p_2 \leq 1$. In this case, the expected consumer surplus for a type-L consumer visiting the single-product brand is simply

$$\begin{cases} v - p_{s_1} & \text{if } p_{s_1} \leq v, \\ 0 & \text{if } p_{s_1} > v, \end{cases}$$

whereas the expected consumer surplus visiting a two-product brand is

$$\begin{cases} \int_0^{v-p_1+p_2} (v - p_1) du + \int_{v-p_1+p_2}^1 (u - p_2) du = \frac{(1-p_2)^2}{2} + \frac{(v-p_1)^2}{2} + p_2(v - p_1) & \text{if } p_1 \leq v, \\ \int_{p_2}^1 (u - p_2) du = \frac{(1-p_2)^2}{2} & \text{if } p_1 > v. \end{cases}$$

A type-L consumer calculates her expected payoff from visiting this leading-1 brand as a weighted sum of the single-product brand with probability β^1 and the two-product brand with probability $1 - \beta^1$ and similarly for other cases with different sets of available brands. She then ranks them in descending order of expected consumer surplus and searches sequentially. She would buy immediately if the realized payoff is positive and higher than the expected consumer surplus from the next brand minus search cost c, continue search if the expected gain in consumer surplus from the next brand minus c is larger than 0, and leave the market without buying if both payoffs are negative.

Now, we can refine the set of strategies the firm may choose in equilibrium. The choices of product variety and product prices are unobserved to consumers before they search and hence do not affect consumer search decisions directly. The firm's revenue comes from both types of consumers, and the two types of consumers may have different probabilities of visiting a certain brand. For visits to each brand, let α denote the proportion of type-L consumers (with $1 - \alpha$ representing type-R consumers). α is an endogenous variable determined by consumers' search decisions. For simplicity, we use a single α here, although it may vary across brands and scenarios for different equilibria.

First, for product variety, a brand created by a single-product firm can only offer its sole product with this sole product as the leading product. We show in Lemma 1 that by contrast the two-product firm will offer both products for any brand it creates.

Lemma 1. *The two-product firm will offer both products for every brand it creates.*

The product variety is revealed to consumers only after they search. For a two-product firm, offering both products within a single brand increases the likelihood that consumers will find a good match for their heterogeneous preferences, leading to a purchase. Additionally, the firm can raise prices to extract higher profits, as it remains indifferent to which product is ultimately sold. Meanwhile, the firm cannot credibly commit to not offering both products before consumers search, which further reinforces its incentive to raise prices. In most cases, the firm optimally raises prices to (v, v)[12], ensuring that consumers either purchase their unfamiliar product when it holds high value or buy their familiar product at their reservation value. As a result, regardless of the proportion α of type-L consumers, the two-product firm earns profit v conditional on consumer visit and leaves any consumer with surplus

$$\int_v^1 (u - v)du + v \cdot 0 = \frac{(1-v)^2}{2}.$$

For j_1 firm, given the proportion of type-L consumers being α, the expected profit of setting price p conditional on consumer visit is $\alpha p + (1-\alpha)p(1-p)$, with optimal price given by

$$p(\alpha) = \begin{cases} \frac{1}{2} + \frac{\alpha}{2(1-\alpha)} & \text{if } \alpha \leq 1 - \frac{1}{2v}, \\ v & \text{if } \alpha > 1 - \frac{1}{2v}. \end{cases}$$

The unconditional expected profit of the firm is the conditional expected profit multiplied by a constant that is not affected by the choice of p directly but depends on consumers' expectation, similarly for j_2 firm.

Next, regarding branding strategies, we show in Lemma 2 that the firm will not create duplicate brands—that is, brands with the same product variety and the same leading product.

Lemma 2. *Neither the single-product firm nor the two-product firm will create duplicate brands in equilibrium.*

An immediate consequence of this lemma is that a single-product firm will create a single brand featuring its sole product. Additionally, following Lemma 1 and Lemma 2, a two-product firm will create at most two brands: either one brand featuring both products or two brands that include both products but with different leading products.

In the following analysis, we first examine the subgame equilibrium under the assumption that the platform recommends all available brands. We then demonstrate that the platform would

[12]In the proofs, we discuss the extreme cases that do not arise in equilibrium.

indeed adopt this strategy in the privacy equilibrium. Building on the previous discussions, we focus on two possible equilibrium types: (1) a two-brand (separating) equilibrium, in which the two-product firm establishes two brands with distinct leading products, and (2) a single-brand (pooling) equilibrium, in which the two-product firm establishes just one brand, equally likely to be leading-1 or leading-2. Among all possible single-brand equilibria, we consider the symmetric one, and in the proof of Lemma 3, we show that this is the only feasible equilibrium when each firm is restricted to creating only one brand.

The two-brand equilibrium is the unique equilibrium when the platform chooses full recommendation. Hereafter, we may refer to the two-brand equilibrium as the equilibrium. We summarize this result in the proposition here and describe these two equilibrium types in detail in the following subsections.

Proposition 1. *If the platform chooses full recommendation, in equilibrium, the two-product firm will create two brands, each offering both products but with distinct leading products. The two-product firm will set prices at v for both products in its brands.*

Since the single-product firm only creates a single brand offering its sole product, the two-product firm has the "freedom" to either distinguish itself from single-product types by creating both leading-1 and leading-2 brands or to pool with them by creating just one brand. Given consumer heterogeneity in preferences, a single brand with only one leading product observable to consumers may deter some from searching. By signaling that it offers both products, the two-product firm can attract more consumer visits with a broader expected range of choices and better capitalize on consumer demand. As a result, the two-product firm optimally chooses to create two brands with distinct leading products to credibly convey this information.

3.2 Two-brand equilibrium

In the two-brand equilibrium, three firm types create different portfolios of brands. Naturally, the j_1 firm creates a leading-1 brand and the j_2 firm creates a leading-2 brand, whereas the j_{12} firm creates both brands, each offering both products. Consumers observe the set of brands and rationally expect the product variety a firm can produce and offer in each brand. When both leading-1 and leading-2 brands are available, they rationally expect it is a two-product firm and both products are available in both brands. As discussed above, the j_{12} firm sets $p_1 = p_2 = v$ for products in both brands to maximize its profit. Anticipating this, consumers randomly choose one of the brands to search. The expected consumer surplus from searching a two-product brand is simply $(1-v)^2/2$, which is identical for both consumer types. Consequently, an equal proportion of two consumer types would search the two-product brand, and the j_{12} firm's unconditional profit is given by $v(1-v)^2/(2\bar{c})$.

The single-product firm creates one brand with its single product. Upon observing just one brand, consumers are certain it offers only that specific product prior to their visit. Denote the product price as p^*. Take the j_1 firm as an example. The expected surplus from visiting its brand is $v - p^*$ for type-L consumers, and $(1-p^*)^2/2$ for type-R consumers. Consumers with search costs below these expected surpluses will search the brand, so the proportion of type-L consumers is $\alpha_{j_1} = (v-p^*)/(v-p^*+(1-p^*)^2/2)$. The firm's expected profit of setting price p is

$$\frac{1}{2}\frac{(1-p^*)^2}{2\bar{c}}p(1-p) + \frac{1}{2}\frac{v-p^*}{\bar{c}}p.$$

The firm's optimal product price, p^*, satisfies

$$p^* = \frac{1}{2} + \frac{v-p^*}{(1-p^*)^2} \in (\frac{1}{2}, v). \tag{1}$$

Here, we use the condition that the equilibrium price the firm sets is consistent with consumers' expectations. The j_1 firm balances the profits from two types of consumers. The optimal price when serving only type-L consumers is v, while the optimal price when serving only type-R consumers is $1/2$. Therefore, the optimal price when serving both types lies between these two values.

In the proof of Proposition 1, we show the j_{12} firm has no incentive to deviate by pretending to be a single-product firm. Here, creating two brands does not directly generate additional consumer visits, since consumers would still choose to visit only one of the brands. However, the presence of two distinct brands signals to consumers that the firm is a two-product firm. Combined with the fact that the two-product firm offers both products in each brand, consumers' expectations regarding product variety change, making them more likely to find a match and, hence, increasing the likelihood of a visit on average.

3.3 Single-brand equilibrium

If the two-product firm just creates one brand, different firm types will be pooled together. We describe here a specific symmetric single-brand equilibrium where the two-product firm creates a single brand with both products, equally likely to be a leading-1 or a leading-2 brand. For each leading-i brand, consumers are uncertain whether it provides just product i or both before they visit, and they assign probability $\beta^i = \gamma$ to it being a single-product brand. As described previously, the two-product firm would charge prices $p_1 = p_2 = v$.

Denote the equilibrium price charged by the single-product firm as p^P. Take j_1 firm as an example. Type-L consumers' expected surplus of visiting a leading-1 brand is $\gamma(v-p^P) + (1-\gamma)(1-v)^2/2$, whereas type-$R$ consumers' expected surplus is $\gamma(1-p^P)^2/2 + (1-\gamma)(1-v)^2/2$. The equilibrium price p^P satisfies

$$p^P = \begin{cases} \frac{1}{2} + \frac{\gamma(v-p^P)+(1-\gamma)(1-v)^2/2}{\gamma(1-p^P)^2+(1-\gamma)(1-v)^2} \in (p^*, v), & \text{if } \gamma > 2(1-v), \\ v & \text{if } \gamma \leq 2(1-v). \end{cases} \tag{2}$$

When the probability of the single-product firm, γ, decreases, the single-product firm will set a higher price. If product prices were observable before consumers search, the rationale would be that consumers' decision to visit a leading-1 brand depends more on the pricing of the two-product firm. A higher price by the j_1 firm would increase the profit margin while only slightly reducing demand. However, since consumers do not observe product prices before they search, prices do not affect consumer decisions directly. Instead, the increasing dependence of consumer surplus on the two-product firm benefits type-L consumers more than type-R consumers when deciding to visit a leading-1 brand. As a result, a disproportionately larger share of type-L consumers would visit the leading-1 brand when γ decreases. Consequently, the j_1 firm would place more weight on profit from type-L consumers who are familiar with the product 1 and charge a higher price. When γ is sufficiently small, the j_1 firm ultimately charges v for product 1—the optimal price when serving only type-L consumers.

This symmetric single-brand candidate equilibrium serves as a benchmark equilibrium. Specifically, it is the unique equilibrium when the firm is restricted to creating only one brand, i.e., when virtual brands are not available.

Lemma 3. *If virtual brands are not available, the two-product firm would create one brand with both products and choose the leading product with equal probability.*

If the two-product firm chooses the leading product with asymmetric probability, say, product 1 with probability ξ and product 2 with probability $1 - \xi$, leading-1 and leading-2 brands would attract different composition and different amount of consumer traffic. For instance, if $\xi > 1/2$, the leading-2 brand will attract consumer visits and the two-product firm would deviate to always choose the leading-1 brand and set $\xi = 0$, so it cannot be an equilibrium.

In the proof of Proposition 1, we show this single-brand candidate equilibrium does not exist with virtual brands available, because the two-product firm will deviate to create two distinct brands to credibly communicate to consumers their product variety and attract more consumers on average.

3.4 Firm profits and consumer welfare

In this subsection, we compare firm profits and consumer welfare in two ways. First, we analyze the expected surplus consumers derive from the single-product firm versus the two-product firm to understand how this affects their search decisions. Second, we compare the two-brand equilibrium with the single-brand benchmark to examine the impact of having virtual brands on market outcomes.

For the first comparison, it is easy to get the following results.

Corollary 1. *In equilibrium, consumers obtain more surplus from a single-product brand selling their unfamiliar product than from a two-product brand, and more from a two-product brand than from a single-product brand selling their familiar product.*

Take the j_1 firm as an example. The j_1 firm faces demand from both consumer types and can extract surplus more effectively from type-L consumers, who have deterministic preferences. This, in turn, benefits type-R consumers, who have uncertain valuations. On the other hand, the j_{12}

firm is more flexible, as it offers both products to consumers, making it more effective at extracting surplus from type-R consumers.

The second comparison examines welfare outcomes under the two-brand equilibrium versus those under the single-brand benchmark (i.e., without virtual brands).

Proposition 2. *With virtual brands available, the two-product firm increases its own profit while also benefiting both types of consumers and the single-product firm.*

Consumers decide whether to visit a brand before observing its full product variety and prices, except for the identity of the leading product. Without virtual brands, the two-product firm must choose one product as the leading product, causing it to be pooled with the single-product firm. Virtual brands help reduce information asymmetry by signaling to consumers before they search. When consumers observe two brands with distinct leading products, they infer that both products are offered and randomly select one to visit. Given consumers' heterogeneous preferences, this additional information increases the likelihood that the "correct" consumers—those more likely to make a purchase—choose to visit. As a result, by balancing consumer traffic from both types, the two-product firm attracts more visits on average and can extract more surplus from consumers. Interestingly, both consumers and the single-product firm also benefit from the availability of virtual brands. On the one hand, knowing the product variety, consumers are more likely to search when they expect to find a desirable product at a reasonable price. The additional variety information helps them adjust their search intensity—searching more when they anticipate a higher surplus and less when they expect a lower surplus. On the other hand, the single-product firm faces more uneven demand, as disproportionately more unfamiliar consumers search its brand. To accommodate this shift, the single-product firm lowers its price, which increases consumer welfare. In turn, consumers anticipating this price adjustment benefits the single-product firm by attracting higher demand. Ultimately, virtual brands not only expand the total market surplus but also create a scenario where all players—consumers, the single-product firm, and the two-product firm—benefit.

3.5 Platform's recommendation

Given that the two-brand equilibrium is the unique subgame equilibrium when the platform chooses full recommendation, we can examine whether doing so is the platform's best interest. As the platform collects a fixed proportion of the firm's revenue (profit), we can directly compare the industry profit across all firm types. From the preceding results, the industry profit when the platform recommends all available brands is

$$\gamma \frac{{p^*}^2(1-p^*)^2}{4\bar{c}} + (1-\gamma)\frac{v(1-v)^2}{2\bar{c}}. \tag{3}$$

Since the single-product firm creates only one brand, selective recommendation by the platform affects the two-product firm when it creates multiple brands. We consider two classes of recommendation strategies. The first is non-targeted recommendation, where the platform recommends the same set of brands to all consumers. The second is targeted recommendation, where the platform tailors brand recommendations based on consumer types.

In the privacy environment, where the platform lacks information about consumer preference types, only non-targeted recommendation is feasible. In contrast, in the no-privacy environment, the platform can implement both non-targeted and targeted recommendations. We show in Proposition 3 that in the privacy equilibrium, although the platform has the ability to filter the brands available to consumers, it chooses not to do so in equilibrium in order to preserve the informational value of virtual brands. As a result, the two-brand equilibrium with full recommendation emerges as the equilibrium under privacy. By contrast, in the no-privacy environment, the platform adopts targeted recommendations, showing each consumer only one two-product brand based on their type. The existence of multiple brands with different leading products enables such targeting.

3.5.1 Non-targeted recommendation

The platform's non-targeted recommendation can potentially add value by filtering the information available to consumers, thereby influencing the firm's pricing strategies and consumer traffic through their expectations. If the platform randomly displays one of the two brands with equal probability when the two-product firm creates two distinct brands, as established in Lemma 1, the firm would still offer both products in each brand. However, this randomization leads to an equilibrium outcome identical to the single-brand benchmark in Section 3.3. For the platform, industry profit is the weighted sum of the single-product firm's profit and the two-product firm's profit. Proposition 1 shows that both firm types achieve higher expected profits under the two-brand equilibrium than under the single-brand benchmark. Therefore, selective recommendation by the platform reduces total industry profit compared to full recommendation. Similarly, if the platform recommends one of the two brands with unequal probability, it still leads to a lower industry profit, making full recommendation the platform's optimal strategy in equilibrium.

A firm's creation of multiple brands, coupled with the platform's full recommendation, effectively communicates information about the product variety to consumers. This reduction in information asymmetry assists in directing consumer traffic, because it enables consumers with heterogeneous preferences to make more informed decisions about whether to explore a particular brand. Consumers would intensify their search efforts when the likelihood of a purchase is higher, and reduce their search intensity when the opposite is true. Should the platform choose not to recommend all available brands, this value derived from information transmission is compromised. Furthermore, as consumers do not observe product prices prior to their search, platform strategies that selectively recommend products cannot directly stimulate consumer search by pushing the firm to lower prices. Hence, the platform benefits from full recommendation. It is important to note the information channel regarding product variety, as outlined here, remains relevant and effective even in the context of a competitive environment involving multiple firms, as we discuss in Section 5.1.

3.5.2 Targeted recommendation

If the two-product firm creates one leading-1 brand and one leading-2 brand, targeting occurs when the platform recommends different brands to different consumer groups. Targeted recommendation can take two forms: (i) The platform recommends each consumer the two-product brand leading with her familiar product. (ii) The platform recommends each consumer the two-product brand leading with her unfamiliar product. Additionally, the platform may adopt a mixed strategy, assigning probabilities to both targeted and non-targeted recommendations. In the main text, we focus on pure targeted recommendation and assume that the two-product firm sets prices at (v, v) to better illustrate the potential benefits of targeted recommendation. In the proof of Proposition 3, we show how the platform, in equilibrium, can fine-tune its mixed (targeted) recommendation strategies to ensure that the two-product firm still sets prices at (v, v). We also show in some cases, the platform may optimally choose targeted recommendation strategy that induces asymmetric pricing by the two-product firm.

Familiar product: If the platform shows each consumer, the two-product brand leading with her familiar product, consumers cannot tell ex ante whether such a brand also offers their unfamiliar product, and they hold belief $\beta = \gamma/(2-\gamma)$ that it just offers the leading product. For the brand leading with their unfamiliar product, consumers know for sure it just offers this single product. Denote the equilibrium price set by the single-product firm as p^F, given by

$$p^F = \begin{cases} \frac{1}{2} + \frac{\gamma(v-p^F)+(1-\gamma)(1-v)^2}{(2-\gamma)(1-p^F)^2} \in (p^*, v), & \text{if } \gamma > 2 - \frac{1}{\frac{3}{2}-v}, \\ v & \text{if } \gamma \leq 2 - \frac{1}{\frac{3}{2}-v}. \end{cases} \tag{4}$$

Compared with the full recommendation equilibrium, the platform's decision to pool the two-product brand with the single-familiar-product brand reduces consumer traffic to the two-product firm. At the same time, as more consumers are drawn to the potential for wider product variety provided by the two-product firm, the single-product firm gains the flexibility to charge higher prices. However, these higher prices in turn reduce consumer traffic due to higher expected costs. As shown in the proof of Proposition 3, both the single-product firm's profits and the two-product firm's profits are lower under this strategy. Consequently, the platform earns a lower expected revenue compared to full recommendation.

Unfamiliar product: If the platform shows each consumer the two-product brand leading with her unfamiliar product, consumers cannot tell ex ante whether such a brand also offers their familiar product, and hold belief $\beta = \gamma/(2-\gamma)$ that it just offers the leading product. Denote the equilibrium price set by the single-product firm as p^U, given by

$$p^U = \frac{1}{2} + \frac{(2-\gamma)(v-p^U)}{\gamma(1-p^U)^2 + (2-2\gamma)(1-v)^2} \in (p^*, p^P). \tag{5}$$

Compared with the full recommendation equilibrium, the two-product firm gains more consumer traffic through pooling. Similar to the familiar product case, the single-product firm would charge higher prices, which leads to less consumer traffic through higher expected prices and lower profits. However, the two-product firm benefits from more consumer traffic and earns a higher profit. It is shown in the proof of Proposition 3 that the industry profit is higher than the profit in full recommendation equilibrium, so the platform benefits from targeted recommendation.[13]

Combining the analyses of both targeted recommendation strategies, we find that targeted recommendation outperforms full recommendation for the platform when targeting is possible. Therefore, the platform's equilibrium strategy is summarized in Proposition 3.

Proposition 3. *In the privacy equilibrium, the platform chooses the full recommendation and the two-product firm creates two distinct brands with different leading products. In the no privacy equilibrium, the two-product firm still creates two distinct brands, and the platform chooses the targeted recommendation by recommending each consumer segment a distinct brand.*

In the privacy environment, the platform maintains an effective communication channel through full recommendation, allowing consumers to observe two distinct brands with different leading products from the two-product firm.[14] This reduction in information asymmetry results in a Pareto improvement, benefiting consumers, the firm, and by extension, the platform itself.

In the no-privacy environment, where the platform has access to consumer information, there is a second function of virtual brands: the presence of multiple brands with different leading products enables targeting. By pooling a two-product brand with different single-product brands for distinct consumer segments, targeted recommendation enhances consumer engagement. While this strategy risks diverting some visits away from the single-product firm, it increases consumer searches for two-product brands, where consumers are more likely to make a purchase upon visiting. The platform strategically guides consumer search intensity toward firms that offer both products, maximizing profitability.[15] Compared to the single-brand benchmark and two-brand equilibrium under full recommendation, the creation of two distinct brands, combined with targeted recommendation, causes consumer beliefs to diverge before search, influencing subsequent decisions. Importantly, the two-product firm does not benefit from personalized pricing—it actually maintains the same prices as in the two-brand equilibrium and single-brand benchmark. Instead, its advantage arises from consumer belief distortion, which drives more traffic to its brands. In the extension, we explore how targeted recommendation can be further advantageous in a competitive market with multiple firms.

[13]We give an example in the proof of Proposition 3 how the targeted recommendation of the two-product brand leading with a consumer's familiar product together with asymmetric pricing can be optimal for the platform in some cases.

[14]This is similar to what we observe in the Amazon example, where preferences for product attributes like color may not be known by the platform ex ante.

[15]UberEats often recommends one of the brands created by the same restaurant at the top of the recommended list for each consumer.

4 Banning Identical-Menu Virtual Brands

A policy-relevant question we would like to answer is how the platform banning the identical-menu virtual brands may affect the firm, consumers and the platform. Specifically, we consider brands can no longer offer the same product across multiple brands. With the ban, the two-product firm is restricted to either create two brands, each offering a distinct product, or one single brand with both products. If the firm opts for a single-brand strategy, the resulting equilibrium mirrors the single-brand equilibrium analyzed earlier, which is no better than the two-brand equilibrium or the targeted recommendation outcome for the platform.

For the two-distinct-brand case, we consider the subgame under full recommendation, and later show full recommendation is indeed optimal for the platform. With each brand offering only a single product, the leading product tells the exact product variety. In the meantime, the number of brands reveals the firm type to consumers. The single-product firm would operate as in the two-brand equilibrium by setting price to be p^*. For the two-product firm, as the two brands offer distinct products, the search order of each consumer does matter in equilibrium. For instance, if each consumers visits the brand offering her familiar product first, the firm would set the price to be v, extracting all the surplus. Expecting this outcome, she would not search first her familiar-product brand in equilibrium. If each consumer visits the her unfamiliar-product brand first, the firm would set the price to be $1/2$ to maximize profit. The consumer surplus conditional on visit is $1/8$. Consumers indeed search the unfamiliar-product brand first only if $v < 5/8$. Otherwise, consumers would instead search the familiar-product brand first, and the equilibrium breaks down.

The above argument, however, is not complete, because some consumers with low search cost may search the second brand given that these two brands now offer different products. We consider symmetric equilibrium here. Let the two-product firm set prices to be $\hat{p}$ for both products in equilibrium. Consumers would rationally expect this decision, and the expected surplus of visiting the unfamiliar-product brand is $(1-\hat{p})^2/2$, while the expected surplus of visiting the familiar-product brand is $v-\hat{p}$. Consumers would indeed search the unfamiliar-product brand first if

$$\frac{(1-\hat{p})^2}{2} \geq v - \hat{p} \iff \hat{p} > \sqrt{2v-1}.$$

Consumers with high search cost would only search the unfamiliar-product brand and decide to buy or not, and the highest search cost to ever start searching is $c = (1-\hat{p})^2/2$. Consumers with lower search cost will continue searching the familiar-product brand if the realized utility of the unfamiliar product u is not high enough. As consumers know the exact utility before they search the familiar-product brand and rationally expect the equilibrium price, the decision to continue searching the familiar-product brand is the same as buying from it in equilibrium. We assume that if consumers see an off-path price p for the first brand they visit, they will adjust their off-path expectation of the product price of the second brand to be the same price p.[16]

[16]Alternatively it can be interpreted as we are focusing on the symmetric equilibrium that the firm always chooses

When two distinct brands are available, consumers with search cost below $(1-\hat{p})^2/2$ will initiate search and start with their unfamiliar-product brand. For the j_{12} firm, the revenue comes from two sources: those who are particularly fond of the unfamiliar product (high u) and those who are not so fond but have low search cost to continue search and buy from the familiar-product brand. The expected profit for the j_{12} firm to charge price p is

$$\Pr\left[c \leq \frac{(1-\hat{p})^2}{2}\right]\left(\Pr\left[c > v-p \ \&\ u > p \middle| c \leq \frac{(1-\hat{p})^2}{2}\right] + \Pr\left[c \leq v-p \middle| c \leq \frac{(1-\hat{p})^2}{2}\right]\right) p$$
$$= \frac{p}{\bar{c}}\left(\frac{(1-\hat{p})^2}{2} - \left(\frac{(1-\hat{p})^2}{2} - (v-p)\right) p\right).$$

Therefore the optimal price $\hat{p}$ is given by

$$\hat{p} = \frac{1}{2} + \frac{\hat{p}(2v-3\hat{p})}{(1-\hat{p})^2}. \tag{6}$$

To sustain the equilibrium search sequence whereby consumers search their unfamiliar-product brand first, we need $\hat{p} > \sqrt{2v-1}$, which implies $v < 2-\sqrt{2}$ and $\hat{p} \in (2v/3, \sqrt{2}-1)$. We show in the proof of Proposition 4 that the two-product firm, creating two brands, each offering a distinct product, is the equilibrium strategy. Because the profit of the single-product firm stays the same, we can easily show the platform also finds it optimal to choose full recommendation.

Proposition 4. *With ban on identical-menu virtual brands, when $v \leq 2-\sqrt{2}$, the platform chooses full recommendation, while the two-product firm would create two distinct brands, each offering a different product, benefiting itself, the platform, and consumers, in comparison with the equilibrium outcome without the ban.*

When the two-product firm introduces two separate brands and each brand offers a different product, the search sequence matters and consumers would search the brand offering their unfamiliar product first. Although diversification seems to make the search less efficient by increasing the overall search cost for consumers, surprisingly, both the two-product firm's profit and consumers' surplus are higher than those under no-ban equilibrium. With consumers self-selecting to search the unfamiliar-product brand first, the two-product firm optimally sets a lower price. First, consumers have a higher expected surplus from searching their unfamiliar product, and the optimal price facing this demand is lower. Second, additional information about price is revealed after consumers visit one of the two brands created by the two-product firm. The two-product firm can use a lower price to credibly persuade consumers to continue searching the second brand in case the first one does not provide a desirable option. However, this equilibrium is only possible when the utility of the familiar product is not so high that consumers are willing to search the brand offering their unfamiliar product first. Without the ban, a "hold-up" problem arises because consumers do not observe product variety and actual product prices prior to their search, while the firm has

the same price for both products. The reasoning is that each brand would attract one of the two consumer segments with equal population to search first. If the two-product firm finds it profitable to choose a off-path price p for one of the brands, it should also be profitable for the other brand.

incentives to raise prices conditional on being searched, which makes the search intensity inefficiently low. The ban provides a commitment device that pushes different brands to offer distinct products and makes the order in which consumers search among these brands pivotal. This change motivates the multi-product firm to lower prices even when consumers cannot observe them prior to their search, which, in turn, increases consumer traffic and mitigates the hold-up problem. By making it hard for the firm to exploit consumers, the ban aligns the interests of consumers and the firm, fostering a win-win situation.

To consider the full equilibrium, we need to check the platform's incentives and also solve the equilibrium when $v > 2 - \sqrt{2}$. So far, we have analyzed the equilibrium where the platform chooses full recommendation. Alternatively, the platform may show only one of the brands when the two-product firm creates two brands. Three general cases exist: (i) The platform targetedly shows consumers their familiar-product brand; (ii) the platform targetedly shows consumers their unfamiliar-product brand; and (iii) the platform, without targeting, randomly shows one of the brands to each consumer with equal probability.[17] We show in Proposition 5 that the platform would choose full recommendation when $v \leq 2 - \sqrt{2}$ and the two-product firm would just create one brand when $v > 2 - \sqrt{2}$ (so that the platform would just recommend the only available brand).

Proposition 5. *When identical-menu virtual brands are banned, the platform would choose full recommendation. If $v \leq 2 - \sqrt{2}$, the two-product firm would create two distinct brands with different products; if $v > 2 - \sqrt{2}$, the two-product firm would just create one brand with both products, equally likely to be leading-1 or leading-2.*

The ban of identical-menu virtual brands together with full recommendation by the platform maintains the informational value of virtual brands by showing product variety to consumers prior to search. In fact, the benefits of recommending all available brands go beyond such informational value because now the two brands offer distinct products and consumers sometimes get material benefit when searching the second brand. At the same time, the two-product firm would lower prices to persuade consumers to continue search. Therefore, not selectively recommending brands is in the platform's best interest. As we said, such a win-win situation only occurs if v is relatively small. Otherwise, consumers would search the brand offering their familiar product first and the firm would want to raise prices, making the equilibrium collapse. When $v > 2 - \sqrt{2}$, the two-product firm would create only one brand with both products and the platform would recommend this single brand. Potentially, the two-product firm can still create two brands with distinct products if the platform targetedly recommends an unfamiliar-product brand to each consumer. This targeted recommendation can solve the hold-up problem and attracts more consumers to visit, but then the two-product firm would be less able to turn visits into purchase with just one product for each consumer. Therefore, when v is relatively large, the ban would turn the equilibrium into the single-brand equilibrium, which is worse for both consumers and the firm.

[17] We already show in the proof of Lemma 3 that the equal-probability randomization is the only possible non-targeted pooling equilibrium.

5 Extensions

We consider several extensions for our main model here and show how the informational channel of product variety through virtual brands plays out in other settings and how some new insights emerge. Proofs of results presented in this section are provided in the Online Appendix.

5.1 Competing firms

We can extend our results to competition among firms. To get rid of the returning demand, assume there are infinitely many firms.[18] Following the sequential search literature, assume search cost c is constant across all consumers and low enough for consumers to initiate search. Here the assumption that consumers know the ownership of each brand is not necessary. For each leading-brand type, infinitely many brands are created by different firms. Information can be communicated through the platform's selective recommendation to consumers instead of being directly communicated through observing multiple brands from the same firm. For example, if two-product firms create two brands with distinct leading products, the platform can observe these different brands and select a list of two-product brands offered by two-product firms. In addition, it can commit to displaying for each consumer a list of two-product brands with the leading product being her familiar product only. Thus, consumers will still know they are searching a list of two-product brands even if they just see one brand from each firm.

To simplify our analysis, we restrict the platform to present either one brand, representing for monopoly, or a list of infinite brands, representing for competition. For the choice of brands, we restrict the platform to choosing just one type of brand for each consumer. If one brand type offers the highest expected profit, the platform would just choose that type for all brands it presents. We focus on the symmetric equilibrium where the same type of firms use the same strategy.

If the platform recommends to consumers just one brand or a list of brands offering only their familiar product, the firms would all charge prices v and no consumer would search for any $c > 0$ as in Diamond (1971). If each consumer is presented a list of brands offering only an unfamiliar product, then for a consumer with search cost c, the optimal strategy is to buy immediately when the value of the product $u > 1 - \sqrt{2c}$, and continue search otherwise. The firms would charge price $\sqrt{2c}$ for their products, with expected profit $2c$. For the platform, the industry profit is $\sqrt{2c}$. If each consumer is presented a list of brands offering both products, with the same leading product, firms can charge different prices for the two products. Without loss of generality, we consider type-L consumers, and leading-1 brand. In the symmetric equilibrium, all these firms charge the same price $p_1^\infty = \frac{5v-1}{4} + \frac{2c}{4(1-v)}$ for product 1 and the same price $p_2^\infty = \frac{3+v}{4} - \frac{2c}{4(1-v)}$ for product 2 in their leading-1 brands. Type-L consumers would buy product 1 if $u < 1 - \frac{4c}{1-v}$ and product 2 otherwise from the first brand she randomly chooses from the list. This equilibrium is only feasible when $c < (1-v)^2/8$. Otherwise, the equilibrium will be that $p_1^\infty = v$, $p_2^\infty = (1+v)/2$, and consumers

[18] Consumers can never exhaust the list of brands, so when they decide to continue search, they find it optimal not to come back and buy from a previously searched brand. See also Jiang and Zou (2020) and Zhong (2023).

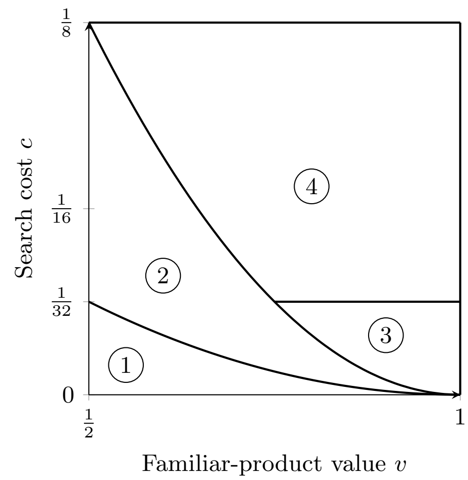

Figure 4: Equilibria with competing firms

Note: The numbered labels in the graph refer to the cases of equilibria in Proposition 6.

would not search any brand, expecting non-positive surplus with positive search cost.[19]

To find the full equilibrium, we focus on four types of strategies for the platform: (1) recommending just one brand offering unfamiliar product only, (2) recommending just one brand offering both products from the same firm, (3) recommending a list of brands offering unfamiliar products only, and (4) recommending a list of brands offering both products.[20] We have the following proposition.

Proposition 6. *The equilibria for different combinations of v and c are as follows.*

1. *When $c < (1-v)^2/8$, the platform would present a single brand offering both products with different leading products targeting different consumer segments (targeted recommendation).*
2. *When $(1-v)^2/8 \leq c \leq (1-v)^2/2$, the platform would present a single brands offering both products with a random leading product for all consumers (non-targeted recommendation).*
3. *When $(1-v)^2/2 < c \leq 1/32$, the platform would present a single brand offering only consumers' unfamiliar product (targeted recommendation).*
4. *When $\max\{(1-v)^2/2, 1/32\} < c < 1/8$, the platform would present a list of brands offering only consumers' unfamiliar product (targeted recommendation).*

In comparison with our main model featuring a single firm, the variety information still plays a pivotal role. The result that two-product firms would create two brands with distinct leading products still holds in most cases with competition. The difference is that the information regarding product variety conveyed through multiple brands can be mediated by the selective recommendation of the platform. When the platform commits to recommending two-product brands, consumers

[19] The analyses are presented in the proof of Proposition 6.

[20] Other possible equilibria may generate the same payoffs as these equilibria under some conditions. In case (4), we focus on targeted recommendation so that each consumer segment receives a (list of) specific brand(s). We use these equilibria to illustrate our key points.

observing just one brand created by a two-product firm can still infer the product variety. When the search cost is low enough, the creation of such virtual brands together with the selective recommendation increases firm and industry profit through better product matching and more sophisticated pricing strategies. As a result, whenever possible, the platform would recommend two-products brands created by two-product firms. In addition, the platform's targeted recommendation based on consumer preference type can facilitate better profit extraction from consumers. Thus, when the search cost is low and consumers would initiate search anyway, targeted recommendation brings more profit for the platform.

The extension also differs with our main model in that we assume the search cost is fixed. Competition inevitably lowers the industry profit, so with a fixed search cost, the platform would recommend to each consumer brands from a single firm only instead of from a list of firms to maximize profit if possible. However, we would expect that when search cost is random, competition may enhance industry profits and be beneficial to the platform by attracting consumers with a higher search cost to search.

With continuous firms, the ban on identical-menu brands essentially breaks the two-product brands created by two-product firms into single-product brands. In absence of uncertainty about the product variety in any brand, the ban would play a similar role as providing consumers a list of single-product brands. When the search cost is constant, the ban cannot encourage more consumer visits and hence would not change platform strategies and the equilibrium. If the search cost is random as in our main model, we can expect the ban to encourage more consumer visits and push down product prices, which could be beneficial to the platform, when it is optimal for the platform to recommend brands from a two-product firm.

5.2 Inferior familiar product

In the main model, we restrict the familiar product to be superior to the unfamiliar product in terms of expected consumer valuation, that is, $v > 1/2$. To complement our main analysis, this section studies the case in which $v < 1/2$; that is, the familiar product offers consumers a lower expected utility than their unfamiliar product. We start with analyzing the firm strategies. An immediate result is that the optimal product prices chosen by the single-product firm would either be v or $1/2$. The j_1 firm, given a fraction α of consumers being type-L, would choose p_1 to maximize

$$\begin{cases} \alpha p_1 + (1-\alpha)(1-p_1)p_1 & \text{if } p_1 \leq v, \\ (1-\alpha)(1-p_1)p_1 & \text{if } p_1 > v. \end{cases}$$

We have the optimal price given by

$$\begin{cases} v & \text{if } \alpha \geq 1 - \frac{4v}{1+4v^2}, \\ \frac{1}{2} & \text{if } \alpha < 1 - \frac{4v}{1+4v^2}. \end{cases}$$

The fraction α is endogenously determined by the search decisions of consumers. Because the familiar product has a lower value v, conditional on consumer visit, the single-product firm would either serve both types of consumers by setting price to be v or charge the monopoly price $1/2$ serving only consumers unfamiliar with it. As consumers cannot earn non-positive surplus from buying their familiar product in any case, they would not visit a brand if for sure it provides just their familiar product. The expected surplus of a consumer visiting a brand with only her unfamiliar product is just $1/8$. Next, by solving the equilibrium and comparing the firm profit under different cases, we have the two-product firm's equilibrium pricing strategy in the separating equilibrium under full recommendation given in the following lemma.

Lemma 4. *In the two-brand equilibrium, the two-product firm would create two brands with distinct leading products. The optimal pricing strategy is to charge both brands* $(1/2, 1/2)$ *if* $v \leq 1 - \sqrt{6}/3$*, charge both* $(v, (1+v)/2)$ *and* $((1+v)/2, v)$ *with equal probability if* $1 - \sqrt{6}/3 < v \leq 1/3$*, and charge both* (v, v) *if* $1/3 < v < 1/2$.

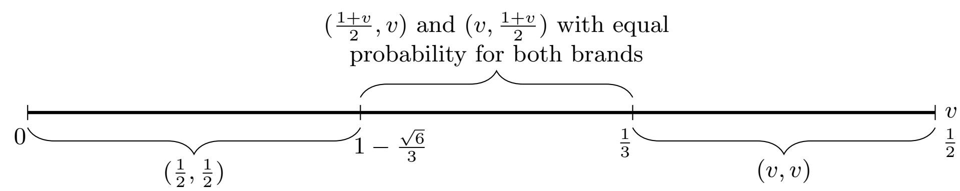

Figure 5: Pricing strategies of the two-product firm with inferior familiar product

In the two-brand equilibrium, the single-product firm would charge price $1/2$ for its sole product and only consumers unfamiliar with this product would visit its brand. By contrast, consumers always have a higher expected surplus from visiting a two-product brand. Therefore, the two-product firm has no incentive to mimic the single-product firm by creating just one brand. We can extend our previous results here that the two-product firm would create two brands with distinct leading products to show its product variety to consumers. The creation of multiple brands increases both its own profit and consumer surplus. For the full equilibrium, we have the following proposition.

Proposition 7. *In equilibrium under* $v < 1/2$*, the platform would choose targeted recommendation if* $v \in (0, 1 - \sqrt{6}/3)$ *and* $\gamma \in ((2 - v^2)/(3 - v^2), 1)$*, and choose full recommendation otherwise. The two-product firm would create two brands with different leading products. By creating virtual brands, the two-product firm increases its own profit while benefiting both types of consumers.*

When the familiar product is inferior to the unfamiliar product, we extend our main finding that the two-product firm would create two brands with different leading products to communicate information about its product variety. This reduction in information asymmetry also benefits consumers so that they search more when they are more likely to buy. In most cases, the platform would also facilitate such communication to guide consumers to search more when they are more likely to purchase.

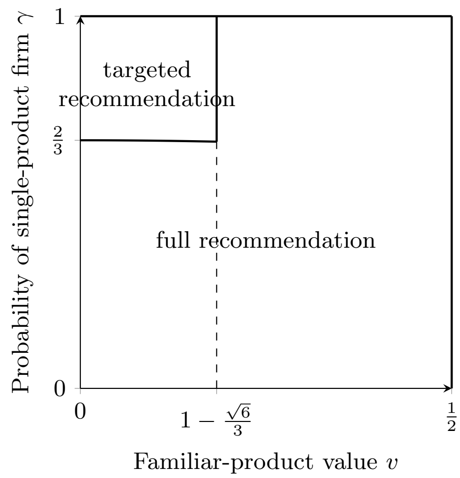

Figure 6: Equilibria under inferior familiar product ($v < 1/2$)

The ban, however, would not benefit the two-product firm nor consumers with inferior familiar product. In the main model with $v > 1/2$, consumers will first search the brand with their unfamiliar product and their self-selection "forces" the firm to charge a lower price. In addition, the firm can use a lower price to persuade consumers to continue searching the second brand and buy the familiar product if the unfamiliar product is not a good match. With $v < 1/2$, however, the unfamiliar product offers a higher expected value and a higher monopoly price than serving just the familiar product. When the identical-menu virtual brands are banned, the two-product firm would just cater the needs of unfamiliar consumers and set price to $1/2$ for each product. Consumers would just search the brand with their unfamiliar product. The ban would not lower the product prices and hence cannot induce more consumer traffic. As a result, the platform cannot use the ban to achieve better equilibrium profits.

5.3 Alternative incentive scheme of the platform

In our model, the platform charges a percentage fee on the total revenue, so the platform shares with the firm the incentive to maximize firm profit, with the mild difference being that the platform tries to maximize the industry profit across all firm types. Another common scenario is that the platform charges a fixed fee per transaction, so its objective is to maximize the total trade volume. We show our results extend to this alternative incentive scheme of the platform.

Proposition 8. *When the platform charges a fixed fee per transaction instead of a percentage fee, the equilibria stay the same. The two-product firm creates two brands with the same menu but distinct leading products; the platform chooses full recommendation in the privacy equilibrium, and targeted recommendation in the no-privacy equilibrium. With ban of identical-menu virtual brands, the platform chooses full recommendation, and the two-product firm would create two distinct brands with different products when* $v \leq 2-\sqrt{2}$*, and would create one brand with both products, equally likely to be leading-1 or leading-2 when* $v > 2-\sqrt{2}$.

Consumers choose whether to visit a brand based on their expected surplus, and hence, the consumer surplus directly determines the transaction volume and affects firm revenue. Therefore, the platform needs to consider both consumer surplus and firm profits under both the total-revenue optimization and transaction-volume optimization. Since the reduction in information asymmetry through virtual brands benefits both consumers and the firm, the platform would still facilitate such information communication by presenting all brands to all consumers when targeting is not possible. Similar to before, targeting is better at extracting surplus when the two-product firm creates two brands with distinct leading products and the platform uses them to target consumers.

6 Conclusion

On online marketplaces and digital platforms, the brand management strategies keep evolving. Companies are increasingly diversifying their online footprint by establishing multiple virtual brands, often offering identical menu of products. Our research studies how creating multiple virtual brands may open up a novel informational channel informing consumers about the firm's product variety, which differs from the traditional umbrella-branding mechanism of product quality signaling.

We develop a model in which a firm with uncertain product variety manages multiple brands. A pivotal discovery of our research is that virtual brands can act as a crucial communication tool for multi-product firms, effectively broadcasting the variety of their product range. The resulting decrease in information asymmetry between the firm and consumers empowers consumers to refine their search strategies, intensifying their search efforts when the expected surplus and purchase likelihood is high. Surprisingly, we found that this dissemination of information benefits not only consumers but also the single-product firm that does not employ such strategies.

In addition, our exploration of the policy prohibiting identical-menu virtual brands suggests that it could lead to a Pareto improvement for both the firm and consumers by inducing more consumers to search brands on the platform. The ban may push different brands to offer distinct products and makes the order in which consumers search among these brands pivotal. This change motivates the multi-product firm to lower prices even when consumers cannot observe them prior to their search, which, in turn, increases consumer traffic. However, this policy warrants careful consideration, as it may lower overall welfare if it leads firms to consolidate their products under a single brand, thereby closing off this informational channel through virtual brands.

We also highlight the role of the platform in setting its recommendation strategies and how these equilibrium strategies differ between a privacy environment, where the platform does not know the consumer's preference type, and a no-privacy environment, where the platform does have this knowledge. In the privacy equilibrium, the platform would sustain the informational channel of communicating product variety using virtual brands by recommending all brands to all consumers, because it results in a win-win-win equilibrium for the two-product firm, consumers and the single-product firm. In the no-privacy equilibrium, the creation of multiple brands with distinct leading products by the firm also makes targeting possible, and the platform would indeed choose targeted recommendation. The targeted recommendation pools each brand created by the two-product firm with a single-product brand featuring the same leading product. The targeted recommendation takes away some consumer traffic from the single-product firm, but brings more consumer visits to the two-product firm that is better at selling to consumers. The industry profit increases as a result.

Several promising directions for future research emerge when considering brands as a tool for communicating product variety information. First, we show our main insights can be extended to competition with infinite firms in the extension. In an oligopoly market with competing firms and uncertain brand ownership, however, the process of consumer belief updating may become more evolved. The prior knowledge of brand ownership and potential consumer behavioral bias can alter communication channels, impacting consumer search decisions and firm competition. Second, our model's consumer heterogeneity is currently based on which product consumers are familiar with. An alternative scenario could involve both products having uncertain values prior to search, with heterogeneity arising from different consumers perceiving different products to have a higher expected utility. In this setting, targeted recommendations from the platform might enhance welfare due to the stronger force of product matching.

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7 Proofs

Proof of Lemma 1. We prove this lemma in two steps. First, we prove the two-product firm will offer both products if it just creates one brand. Next, we prove the case where the two-product firm creates multiple brands. The trembling hand refinement ensures that each brand introduced by the firm has a nonzero probability of being considered by consumers, and that the firm maximizes its profit given a particular composition of consumers.

In the **single-brand** case, say, a two-product firm creates a leading-1 brand, and a fraction $\alpha \in [0,1]$ of consumers visiting this brand are type-L. The actual choices of product variety and prices are not observed before consumers search, and hence do not directly affect consumers' search decisions. Conditional on this brand being searched, if the firm offers only product 1 in the brand, it chooses product-1 price p to maximize the profit as $\alpha p + (1-\alpha)p(1-p)$. The optimal price $p(\alpha)$ and optimized profit are

$$p(\alpha) = \begin{cases} \frac{1}{2(1-\alpha)} \\ v \end{cases} \qquad \pi(\alpha) = \begin{cases} \frac{1}{4(1-\alpha)} & \text{if } \alpha < 1 - \frac{1}{2v}, \\ v - (1-\alpha)v^2 & \text{if } \alpha \geq 1 - \frac{1}{2v}. \end{cases}$$

If the firm offers both products, denote product-1 and product-2 prices as p, p'. (1) If $p, p' \leq v$, both types of consumers would buy one of the products for sure and the firm maximizes the profit as

$$\begin{aligned} &\alpha\left(p\Pr[v-p \geq u-p'] + p'\Pr[v-p < u-p']\right) + (1-\alpha)\left(p\Pr[u-p \geq v-p'] + p'\Pr[u-p < v-p']\right) \\ =& -(p-p')^2 + (2\alpha-1)v(p-p') - \alpha(p-p') + p. \end{aligned}$$

The optimal product prices are $p = p' = v$ regardless of α, and the optimized profit is v.[21] This profit is higher than the profit from offering just one product. (2) If $p, p' > v$, only consumers unfamiliar with the product would buy and it is optimal to set prices $p = p' = 1/2$, which forms a contradiction. (3) If $p \leq v < p'$, the firm would maximize the profit as

$$\begin{aligned} &\alpha\left(p\Pr[v-p \geq u-p'] + p'\Pr[v-p < u-p']\right) + (1-\alpha)p\Pr[u-p \geq 0] \\ =& -\alpha(p'-p)^2 + \alpha(1-v)(p'-p) - (1-\alpha)p^2 + p. \end{aligned}$$

The optimal prices and maximized profit are

$$(p(\alpha), p'(\alpha)) = \begin{cases} \left(\frac{1}{2(1-\alpha)}, \frac{1}{2(1-\alpha)} + \frac{1-v}{2}\right) \\ \left(v, \frac{1+v}{2}\right) \end{cases} \qquad \pi(\alpha) = \begin{cases} \alpha\frac{(1-v)^2}{4} + \frac{1}{4(1-\alpha)} & \text{if } \alpha < 1 - \frac{1}{2v}, \\ \alpha\frac{(1+v)^2}{4} + (1-\alpha)v(1-v) & \text{if } \alpha \geq 1 - \frac{1}{2v}. \end{cases}$$

This profit is also higher than the profit of offering just one product. Therefore comparing the profits from these 3 possible strategies, the two-product firm will offer both products if it creates

[21] For the equality, it is easy to see $v - p + p' > 0$, and verify the profit is increasing in p if $v - p + p' > 1$. Therefore, $v - p + p', v - p' + p \in [0,1]$.

just one brand and have the optimal pricing strategies given by[22]

$$\begin{cases} (v, v) & \text{if } \frac{(1-v)^2}{4v^2+(1-v)^2} \leq \alpha \leq \frac{4v^2}{4v^2+(1-v)^2}, \\ (v, \frac{1+v}{2}) & \text{if } \alpha > \frac{4v^2}{4v^2+(1-v)^2} > 0.8, \\ (\frac{1+v}{2}, v) & \text{if } \alpha < \frac{(1-v)^2}{4v^2+(1-v)^2} < 0.2. \end{cases}$$

In the **multi-brand** case, consider a consumer searches a brand $B1$ and decides whether to search the next brand $B2$. If both $B1$ and $B2$ just offer product 1, the consumer would just search first the brand with lower expected price and not search the second brand. If $B1$ just offers product 1 at price p_1 and $B2$ just offers product 2 at price p_2, the firm would make a higher profit by offering both products in both $B1$ and $B2$ at prices (v, v). Since consumers do not observe product variety beyond the leading product and product prices, this would not change consumers' first search decision, and consumers will pay v for sure conditional on searching $B1$ as $B2$ offers the same set of products at the same prices. If $B1$ just offers one product and $B2$ offers both products or vice versa, similar arguments hold. The same argument still holds if there are more brands, because consumers do not observe product variety and product prices, and the same product offered in different brands provide consumers with the same value.

Therefore, the firm would offer both products in any brand it creates. □

Proof of Lemma 2. Consumers decide whether and which brand to search without knowing the product variety in each brand, except for the leading product. First, consider a candidate equilibrium in which the two-product firm j_{12} creates two duplicate brands—specifically, two leading-1 brands. In this case, consumers cannot directly determine whether product 2 is available in these brands or whether the brands are offered by a single-product firm j_1 or a two-product firm j_{12}. Instead, they must infer this information based on the equilibrium strategies played by the firm and form beliefs accordingly.

(1) Separating equilibrium where the sets of brands created by the j_1 firm and j_{12} firm are different: If the j_1 firm attracts more consumer traffic, the j_{12} firm can easily mimic j_1's branding strategies and earn a higher profit; conversely, if the j_{12} firm attracts more consumer traffic, the j_1 firm can similarly mimic j_{12}'s branding strategies and earn a higher profit. If j_1 and j_{12} receive the same consumer traffic, j_{12} can deviate to create just one leading-1 brand. Given the negligible cost of creating each brand, the intuitive criterion implies that j_{12} can earn at least the same profit plus cost savings of one additional brand and would therefore deviate.

(2) Pooling equilibrium where the sets of brands created by the j_1 firm and j_{12} firm are the same: Either the j_1 firm or the j_{12} firm can deviate by creating just one leading-1 brand. Given

[22]Notice, however, that the asymmetric pricing is hard to sustain in equilibrium, because the fraction α depends on consumers' expectation of surplus. When the prices are set as $(v, (1+v)/2)$, type-L consumers would expect lower consumer surplus than type-R consumers, so the resulting fraction α would be small, making the prices $(v, (1+v)/2)$ non-optimal, similarly for $((1+v)/2, v)$. The asymmetric pricing may occur when the platform uses targeted recommendation, which we discuss later.

the negligible cost of creating each brand, the intuitive criterion implies that either firm can earn at least the same profit plus cost savings of one additional brand by deviating.

Therefore, the candidate equilibrium cannot be sustained as an equilibrium. Similar arguments apply to cases where the j_1 or j_2 firm creates duplicate brands. In sum, creating duplicate brands does not generate additional value and cannot serve as a credible communication device. Hence, it cannot be an equilibrium strategy for the firm. □

Proof of Lemma 3. **(1) Symmetric pricing:** Suppose the two-product firm sets prices (v, v). First consider a semi-pooling case where j_{12} firm creates only a leading-1 brand. Consumers are certain that a leading-2 brand is created by j_2 firm and only provides product 2 before they search. j_2 firm faces the same problem as in the two-brand equilibrium and would set $p_2 = p^*$ with the same expected profit. Let the equilibrium price set by j_1 firm be p^{SP}. For a leading-1 brand, from consumers' perspective, the probability of it being a single-product brand is $\beta^1 = \gamma/(2-\gamma)$, so the expected payoff is $\frac{\gamma}{2-\gamma}(v - p^{SP}) + \frac{2-2\gamma}{2-\gamma}\frac{(1-v)^2}{2}$ for type-L consumers , and $\frac{\gamma}{2-\gamma}\frac{(1-p^{SP})^2}{2} + \frac{2-2\gamma}{2-\gamma}\frac{(1-v)^2}{2}$ for type-R consumers. The j_1 firm sets the equilibrium price

$$p^{SP} = \begin{cases} \frac{1}{2} + \frac{\gamma(v-p^{SP})+(1-\gamma)(1-v)^2}{\gamma(1-p^{SP})^2+(2-2\gamma)(1-v)^2} \in (p^P, v), & \text{if } \gamma > \frac{4-4v}{3-2v}, \\ v & \text{if } \gamma \le \frac{4-4v}{3-2v}. \end{cases}$$

When the proportion of single-product firm γ decreases, it sets a higher price based on the same reasoning as in the symmetric single-brand equilibrium in the main text.

The semi-pooling and symmetric pooling equilibria are special cases of more general single-brand equilibria where the two-product firm creates one brand, with probability $\eta \in [0, 1]$ being leading-1. The semi-pooling case is $\eta = 1$ and the symmetric pooling one is $\eta = 1/2$. Given η, consumers assign probability $\beta^1 = \gamma/(\gamma + 2(1-\gamma)\eta)$ to a leading-1 brand selling just product 1 and probability $\beta^2 = \gamma/(\gamma + 2(1-\gamma)(1-\eta))$ to a leading-2 brand just selling product 2. Denote the price charged by j_i firm as $\tilde{p}_i$, $i \in \{1, 2\}$. The expected surplus of visiting a leading-i brand is $\beta^i(v-\tilde{p}_i)+(1-\beta^i)\frac{(1-v)^2}{2}$ for type-L consumers, and $\beta^i\frac{(1-\tilde{p}_i)^2}{2}+(1-\beta^i)\frac{(1-v)^2}{2}$ for type-R consumers. The equilibrium price $\tilde{p}_i$ satisfies

$$\tilde{p}_i = \begin{cases} \frac{1}{2} + \frac{\beta^i(v-\tilde{p}_i)+(1-\beta^i)(1-v)^2/2}{\beta^i(1-\tilde{p}_i)^2+(1-\beta^i)(1-v)^2} & \text{if } \beta^i > 2(1-v), \\ v & \text{if } \beta^i \le 2(1-v). \end{cases}$$

It is easy to show that $\tilde{p}_i$ is decreasing in β^i. For the two-product firm, the expected profit of mixing with probability η is

$$\frac{\eta}{2\bar{c}}\left(\beta^1(v-\tilde{p}_1) + \beta^1\frac{(1-\tilde{p}_1)^2}{2} + (1-\beta^1)(1-v)^2\right)v + \frac{1-\eta}{2\bar{c}}\left(\beta^2(v-\tilde{p}_2) + \beta^2\frac{(1-\tilde{p}_2)^2}{2} + (1-\beta^2)(1-v)^2\right)v.$$

It can be shown that $\beta^i(v-\tilde{p}_i) + \beta^i\frac{(1-\tilde{p}_i)^2}{2} + (1-\beta^i)(1-v)^2$ is strictly increasing in β^i and β^1 (β^2) is strictly decreasing (increasing) in η, so the two terms in the big parentheses are equal only when $\eta = 1/2$. As the expected profit is linear in η, the optimal η must be 0 or 1 or 1/2. However, if $\eta = 0$, then the first term is larger, meaning that a leading-1 brand has more consumer visits, so the optimal η should be 1. Similarly for $\eta = 1$. Therefore, only the symmetric single-brand equilibrium ($\eta = 1/2$) can sustain.

(2) Asymmetric pricing: Now consider the two-product firm chooses prices $((1+v)/2, v)$ (similar for the case with $(v, (1+v)/2)$). Keep the notion for β^i and $\tilde{p}_i$. The equilibrium price $\tilde{p}_1$ of j_i firm satisfies

$$\tilde{p}_1 = \begin{cases} \frac{1}{2} + \frac{\beta^1(v-\tilde{p}_1)+(1-\beta^1)(1-v)^2/2}{\beta^1(1-\tilde{p}_1)^2+(1-\beta^1)(1-v)^2/4} & \text{if } \beta^1 > \frac{5-2v}{1+6v}, \\ v & \text{if } \beta^1 \le \frac{5-2v}{1+6v}, \end{cases}$$

where $\tilde{p}_1$ is decreasing in β^1. Similarly we can get the expression for $\tilde{p}_2$. For the two-product firm, the expected profit of mixing with probability η is

$$\frac{\eta}{2\bar{c}}\left(\left(\beta^1(v-\tilde{p}_1)+(1-\beta^1)\frac{(1-v)^2}{2}\right)v+\left(\beta^1\frac{(1-\tilde{p}_1)^2}{2}+(1-\beta^1)\frac{(1-v)^2}{8}\right)\frac{(1+v)^2}{4}\right)$$
$$+\frac{1-\eta}{2\bar{c}}\left(\left(\beta^2\frac{(1-\tilde{p}_2)^2}{2}+(1-\beta^2)\frac{(1-v)^2}{2}\right)v+\left(\beta^2(v-\tilde{p}_2)+(1-\beta^2)\frac{(1-v)^2}{8}\right)\frac{(1+v)^2}{4}\right).$$

In general the two terms in the big parentheses are equal only when $\eta = \tilde{\eta}$ for some $\tilde{\eta} > 1/2$. Therefore, the optimal η must be 0 or 1 or $\tilde{\eta}$. However, if $\eta = \tilde{\eta}$, the two-product firm can deviate to $(v, (1+v)/2)$ for higher profit. In addition, it's better to deviate to $(v, (1+v)/2)$ if $\eta = 0$ as more type-L consumers would visit. If $\eta = 1$, it's better for the two-product firm to deviate to $(v, (1+v)/2)$ and $\eta = 0$, i.e. creating a leading-2 brand instead of a leading-1 brand. Therefore, there is no single-brand equilibrium with asymmetric prices. □

Proof of Proposition 1. **(1)** First we show the candidate two-brand equilibrium is indeed an equilibrium. In this case, the single-product firm cannot deviate and pretend to be a two-product firm. If the two-product firm deviates to create just one leading-1 (or leading-2) brand and sets $(p_1, p_2) = (v, v)$, it earns

$$\frac{1}{2\bar{c}}\left(v - p^* + \frac{(1-p^*)^2}{2}\right)v < \frac{v(1-v)^2}{2\bar{c}}.$$

To see this, we first get from equation (1) that $\frac{\partial p^*}{\partial v} = \frac{1}{3p^{*2}-5p^*+3} > 0$. Therefore,

$$\frac{\partial}{\partial v}\left(v - p^* + \frac{(1-p^*)^2}{2} - (1-v)^2\right) = \frac{\left(2-(2p^*-1)(2-2p^*+p^{*2})\right)(3p^{*2}-5p^*+3)+(2-p^*)}{3p^{*2}-5p^*+3}.$$

As $p^* \in (0.5, v) \subset (0.5, 1)$, the denominator is always positive and the numerator is strictly decreasing in p^* and always positive, so the difference in the profits is strictly increasing. At $v = 1$, the two profits are equal. Therefore, the initial inequality holds. Similarly, it can be shown that if the two-product firm deviates to create just one leading-1 brand and sets $(p_1, p_2) = (v, (1+v)/2)$, it earns

$$\frac{1}{2\bar{c}}\left((v-p^*)\,v(1-v)+\frac{(1-p^*)^2}{2}\frac{(1+v)^2}{4}\right)<\frac{v(1-v)^2}{2\bar{c}}.$$

Thus, the two-product firm has no incentive to deviate and the candidate two-brand equilibrium is indeed an equilibrium.

(2) Next we show that the benchmark single-brand case is not an equilibrium. In the single-brand benchmark, the two-product firm's expected profit is

$$\frac{1}{2\bar{c}}\left(\gamma\left(v-p^P+\frac{(1-p^P)^2}{2}\right)+(1-\gamma)(1-v)^2\right)v.$$

If the two-product firm deviates to create two brands with distinct leading products, consumers would rationally expect both products are available, and the firm's optimal profit, as in the two-brand equilibrium, is $v(1-v)^2/(2\bar{c})$. The two-product firm would earn a higher profit as

$$(1-v)^2>v-p^*+\frac{(1-p^*)^2}{2}>v-p^P+\frac{(1-p^P)^2}{2}.$$

In conclusion, there is an unique equilibrium when the platform chooses full recommendation: the two-brand equilibrium. □

Proof of Proposition 2. The profit of the two-product firm under the two-brand equilibrium is higher because

$$\frac{v}{2\bar{c}}\left(\gamma\frac{(1-p^P)^2}{2}+\gamma(v-p^P)+(1-\gamma)(1-v)^2\right)<\frac{v(1-v)^2}{2\bar{c}}\Leftrightarrow\ v-p^P+\frac{(1-p^P)^2}{2}<(1-v)^2,$$

which always holds. Consumers' expected surplus under the two-brand equilibrium is higher if

$$\left(p^*-p^P\right)\left(2-\frac{p^*+p^P}{2}\right)<0,$$

which always holds. Therefore, consumers also benefit from the presence of virtual brands. Lastly, for the single-product firm, the change in expected profit from two-brand equilibrium to single-brand equilibrium is

$$\frac{{p^P}^2(\gamma(1-p^P)^2+(1-\gamma)(1-v)^2)}{4\bar{c}}-\frac{{p^*}^2(1-p^*)^2}{4\bar{c}}<\frac{{p^P}^2(1-p^P)^2}{4\bar{c}}-\frac{{p^*}^2(1-p^*)^2}{4\bar{c}}<0.$$

Therefore, the single-product firm also benefits from the presence of virtual brands. □

Proof of Proposition 3. In the privacy environment, if the platform chooses non-targeted recommendation of one of the brands, as we argued, the equilibrium outcome will be the same as one of the pooling equilibria in the proof of Proposition 2. We have shown that consumers and all firm types are better off under full recommendation, so is the platform.

Now consider targeted recommendation in no-privacy environment. **(1) Symmetric pricing:** Suppose for now that the j_{12} firm charges price (v, v) for both brands. **(1F)** First we show the platform earns lower profit by targeted recommending each consumer the brand leading with her familiar product if there are two distinct brands. Denote the equilibrium price charged by the single-product firm as p^F,given by equation (4). Compared with the two-brand equilibrium, j_1 firm's profit is lower if

$$\frac{{p^F}^2(1-p^F)^2}{4\bar{c}} < \frac{p^*(1-p^*)^2}{4\bar{c}} \Leftarrow p^F > p^* > \frac{1}{2},$$

which always holds, and j_{12} firm's profit is lower if

$$\frac{\gamma}{2-\gamma}(v-p^F) + \frac{2-2\gamma}{2-\gamma}\frac{(1-v)^2}{2} < \frac{(1-v)^2}{2} \iff v - p^F < \frac{(1-v)^2}{2},$$

which always holds. Therefore, the platform would earn a lower expected profit by adopting this strategy than full recommendation.

(1U) Now we show the platform earns higher profit by targeted recommending each consumer the brand leading with her unfamiliar product if there are two distinct brands. The expected profit for the j_1 firm of charging price p is

$$\frac{1}{2\bar{c}}(v-p^U)p + \frac{1}{2\bar{c}}\left(\frac{\gamma}{2-\gamma}\frac{(1-p^U)^2}{2} + \frac{2-2\gamma}{2-\gamma}\frac{(1-v)^2}{2}\right)p(1-p),$$

so the optimal price p^U is given by equation (5). Taking the first order derivative with respect to γ, we have

$$\frac{\partial p^U}{\partial \gamma} = \frac{\frac{v-p^U}{2p^U-1} + \frac{(1-p^U)^2}{2} - (1-v)^2}{\gamma(1-p^U) + (2-\gamma)\frac{1-2v}{(2p^U-1)^2}} = \frac{1}{2-\gamma}\frac{(1-p^U)^2-(1-v)^2}{\gamma(1-p^U)+(2-\gamma)\frac{1-2v}{(2p^U-1)^2}}.$$

The numerator is obviously positive. For the denominator, the partial derivative with respect to p^U equals

$$-\gamma - (2-\gamma)\frac{4(1-2v)}{(2p^U-1)^3} > -\gamma + (2-\gamma)\frac{4}{(2v-1)^2} > 0.$$

Therefore, the denominator

$$\gamma(1-p^U) + (2-\gamma)\frac{1-2v}{(2p^U-1)^2} < \gamma(1-v) - \frac{2-\gamma}{2v-1} < 0.$$

This means that p^U is decreasing in γ. Taking the second order derivative with respect to γ, we have

$$\frac{\partial^2 p^U}{\partial \gamma^2} = \frac{\left((2-\gamma)\frac{4(1-2v)}{(2p^U-1)^3} + \gamma\right)\left(\frac{\partial p^U}{\partial \gamma}\right)^2 + 2\left(\frac{1-2v}{(2p^U-1)^2} - (1-p^U)\right)\frac{\partial p^U}{\partial \gamma}}{\gamma(1-p^U) + (2-\gamma)\frac{1-2v}{(2p^U-1)^2}}.$$

Compared with the separating equilibrium, j_1 firm's profit is lower while the two-product firm's profit is higher.For the platform, under separating equilibrium, the industry profit is expression (3); under the targeted recommendation of unfamiliar product, the industry profit is

$$\frac{1}{\bar{c}}\left(\frac{\gamma}{2-\gamma}\frac{(1-p^U)^2}{2}+\frac{2-2\gamma}{2-\gamma}\frac{(1-v)^2}{2}\right)\left(\gamma\frac{p^{U2}}{2}+(1-\gamma)v\right). \tag{7}$$

It is easy to see that the above two expected profits are the same when $\gamma \in \{0,1\}$. The partial derivative of the industry profit (3) in separating equilibrium with respect to γ is constant at

$$\frac{{p^*}^2(1-p^*)^2}{4\bar{c}}-\frac{(1-v)^2}{2\bar{c}}v<0.$$

The first order partial derivative of the industry profit (7) in targeted recommendation with respect to γ is

$$\frac{1}{\bar{c}}\left(\frac{p^{U2}}{2}-v\right)\frac{v-p^U}{2p^U-1}+\frac{1}{\bar{c}}\left(\left(\gamma\frac{p^{U2}}{2}+(1-\gamma)v\right)\frac{1-2v}{(2p^U-1)^2}+\gamma p^U\frac{v-p^U}{2p^U-1}\right)\frac{\partial p^U}{\partial\gamma}.$$

The second order partial derivative of the targeted industry profit with respect to γ is

$$\frac{1}{\bar{c}}\frac{A_1\frac{\partial p^U}{\partial\gamma}+A_2\left(\frac{\partial p^U}{\partial\gamma}\right)^2}{\gamma(1-p^U)+(2-\gamma)\frac{1-2v}{(2p^U-1)^2}},$$

where the denominator has been shown to be negative, and

$$\begin{aligned}
A_1 =&\frac{2(2v-1)}{(2p^U-1)^4}\left((1-v)p^U(2p^U-1)^2-(v-p^U)(2p^U-1)-(v-p^{U^2})(2v-1)\right)\\
<&\frac{2(2v-1)}{(2p^U-1)^3}\left((1-v)p^U(2p^U-1)-2v+p^U+p^{U^2}\right)<0,\\
A_2 =&\frac{\gamma(2v-1)}{(2p^U-1)^3}\left((2-\gamma)(3p^U-v)-(2p^U-1)\left(2\gamma p^U-\frac{3}{2}\gamma p^{U^2}+(1-\gamma)v\right)\right.\\
&\left.+4(1-p^U)\left(\gamma\frac{p^{U2}}{2}+(1-\gamma)v\right)\right)+\gamma^2\frac{v-p^U}{2p^U-1}>0.
\end{aligned}$$

The signs of A_1, A_2 can be shown by first showing the terms in the big parentheses are increasing in p^U. Hence $A_1\frac{\partial p^U}{\partial\gamma}+A_2\left(\frac{\partial p^U}{\partial\gamma}\right)^2>0$. Therefore the first order partial derivative of the selective recommendation profit is strictly decreasing in γ while the first order partial derivative of the separating equilibrium profit is constant, so the difference between the targeted industry profit and the separating industry profit must be first increasing and then decreasing in γ. Hence the difference is always above 0 as the differences at the two end points are 0.

(2) Asymmetric Pricing: Now we show the platform would earn higher profit if the two-product firm chooses prices (v, v) instead of asymmetric pricing under targeted recommendation.

With targeted recommendation of the two-product brand leading with unfamiliar product, the j_{12} firm can charge asymmetric prices, $(1+v)/2$ for the leading product and v for the second product to maximize profit. The expected profit for the j_1 firm of charging price p is

$$\frac{1}{2\bar{c}}(v-p^{U'})p+\frac{1}{2\bar{c}}\left(\frac{\gamma}{2-\gamma}\frac{(1-p^{U'})^2}{2}+\frac{2-2\gamma}{2-\gamma}\frac{(1-v)^2}{8}\right)p(1-p),$$

so the optimal price $p^{U'}$ is given by

$$p^{U'}=\frac{1}{2}+\frac{v-p^{U'}}{\frac{\gamma}{2-\gamma}(1-p^{U'})^2+\frac{2-2\gamma}{2-\gamma}\frac{(1-v)^2}{4}}\in(p^U,v). \tag{8}$$

When $\gamma=1$, $p^{U'}=p^U=p_1^*$. Taking the first order derivative with respect to γ, we have

$$\frac{\partial p^{U'}}{\partial\gamma}=\frac{\frac{v-p^{U'}}{2p^{U'}-1}+\frac{(1-p^{U'})^2}{2}-\frac{(1-v)^2}{4}}{\gamma(1-p^{U'})+(2-\gamma)\frac{1-2v}{(2p^{U'}-1)^2}}=\frac{1}{2-\gamma}\frac{(1-p^{U'})^2-\frac{(1-v)^2}{4}}{\gamma(1-p^{U'})+(2-\gamma)\frac{1-2v}{(2p^{U'}-1)^2}}.$$

The numerator is obviously positive. The denominator, as before, can be shown to be negative. This means that p^U is decreasing in γ. Taking the second order derivative with respect to γ, we have

$$\frac{\partial^2 p^{U'}}{\partial\gamma^2}=\frac{\left((2-\gamma)\frac{4(1-2v)}{(2p^{U'}-1)^3}+\gamma\right)\left(\frac{\partial p^{U'}}{\partial\gamma}\right)^2+2\left(\frac{1-2v}{(2p^{U'}-1)^2}-(1-p^{U'})\right)\frac{\partial p^{U'}}{\partial\gamma}}{\gamma(1-p^{U'})+(2-\gamma)\frac{1-2v}{(2p^{U'}-1)^2}}.$$

For the platform, under separating equilibrium, the industry profit is expression (3); under the targeted recommendation of unfamiliar product, the industry profit is

$$\frac{1}{\bar{c}}\left(\frac{\gamma}{2-\gamma}\frac{(1-p^{U'})^2}{2}+\frac{2-2\gamma}{2-\gamma}\frac{(1-v)^2}{8}\right)\left(\gamma\frac{p_1^{U'2}}{2}+(1-\gamma)\frac{(1+v)^2}{4}\right). \tag{9}$$

It is easy to see that the expected profit under targeted recommendation is the same as the profit under the separating equilibrium when $\gamma=0$, and is lower when $\gamma=1$. The first order partial derivative of the industry profit (9) in targeted recommendation with respect to γ is

$$\frac{1}{\bar{c}}\left(\frac{p_1^{U'2}}{2}-\frac{(1+v)^2}{4}\right)\frac{v-p^{U'}}{2p^{U'}-1}+\frac{1}{\bar{c}}\left(\left(\gamma\frac{p_1^{U'2}}{2}+(1-\gamma)\frac{(1+v)^2}{4}\right)\frac{1-2v}{(2p^{U'}-1)^2}+\gamma p^{U'}\frac{v-p^U}{2p^{U'}-1}\right)\frac{\partial p^{U'}}{\partial\gamma}.$$

The second order partial derivative of the targeted industry profit with respect to γ is

$$\frac{1}{\bar{c}}\frac{A_1'\frac{\partial p^{U'}}{\partial\gamma}+A_2'\left(\frac{\partial p^{U'}}{\partial\gamma}\right)^2}{\gamma(1-p^{U'})+(2-\gamma)\frac{1-2v}{(2p^{U'}-1)^2}},$$

where the denominator has been shown to be negative, and similar to above, it can be shown that $A_1' < 0$ and $A_2' > 0$. Hence $A_1 \frac{\partial p^U}{\partial \gamma} + A_2 \left(\frac{\partial p^U}{\partial \gamma}\right)^2 > 0$. Therefore the first order partial derivative of the selective recommendation profit is strictly decreasing in γ while the first order partial derivative of the separating equilibrium profit is constant. In addition, the difference in the first order derivatives at $\gamma = 0$ is negative. Therefore, the difference in the first order derivatives is always below 0 and the platform earns a lower profit than the profit when the two-product firm charges (v, v).

(3) Now we show how the platform can use mixed targeted recommendation to make sure the two-product firm sets prices (v, v). Combining the above analyses, the platform would earn the highest profit when it targetedly recommends the two-product brand leading with consumers' unfamiliar product. However, the two-product firm would deviate to asymmetric pricing if the platform always targetedly recommends this way. The platform can sustain the prices (v, v) by targetedly recommending the two-product brand leading with consumers' unfamiliar product with probability ϕ and recommending both brands with probability $1 - \phi$ when two distinct brands are available. The optimal ϕ satisfies

$$\frac{(1-\phi)\frac{(1-v)^2}{4}}{\phi\left(\frac{\gamma}{\gamma+2(1-\gamma)\phi}\frac{(1-p_1^{U*})^2}{2} + \frac{2(1-\gamma)\phi}{\gamma+2(1-\gamma)\phi}\frac{(1-v)^2}{2}\right) + (1-\phi)\frac{(1-v)^2}{4}} = \frac{(1-v)^2}{4v^2} \tag{10}$$

where p_1^{U*} solves

$$p_1^{U*} = \frac{1}{2} + \frac{v - p_1^{U*}}{\frac{\gamma}{\gamma+2(1-\gamma)\phi}\frac{(1-p_1^{U*})^2}{2} + \frac{2(1-\gamma)\phi}{\gamma+2(1-\gamma)\phi}\frac{(1-v)^2}{2}} \in (p^P, p^U). \tag{11}$$

(4) At last, we show an example in which the platform finds it optimal to targetedly recommend to each consumer the brand leading with their familiar product under asymmetric pricing under some parameters. Consider $v = 0.9$ and $\gamma = 0.1$. The j_{12} firm would optimally choose asymmetric prices, v for the familiar product and $(1 + v)/2$ for the unfamiliar product. Denote the equilibrium price charged by the single-product firm as $p^{F'}$. The expected profit for the j_1 firm of charging price p is

$$\frac{1}{2\bar{c}}\left(\frac{\gamma}{2-\gamma}(v - p^{F'}) + \frac{2-2\gamma}{2-\gamma}\frac{(1-v)^2}{8}\right)p + \frac{1}{2\bar{c}}\frac{(1-p^{F'})^2}{2}p(1-p),$$

and the optimal price can be solved as $p^{F'} \approx 0.608$. For the platform, the industry profit is given by

$$\gamma\frac{p^{F'2}(1-p^{F'})^2}{4\bar{c}} + (1-\gamma)\frac{1}{\bar{c}}\left(\frac{\gamma}{2-\gamma}(v - p^{F'}) + \frac{2-2\gamma}{2-\gamma}\frac{(1-v)^2}{2}\right)v \approx \frac{0.0177}{\bar{c}}.$$

When the platform chooses full recommendation, the equilibrium price charged by the single-product firm is $p^* \approx 0.896$, and the industry profit is given by

$$\gamma\frac{{p^*}^2(1-p^*)^2}{4\bar{c}}+(1-\gamma)\frac{(1-v)^2}{2\bar{c}}v\approx\frac{0.0177}{\bar{c}}\approx\frac{0.00427}{\bar{c}}.$$

When the platform chooses optimal targeted recommendation of two-product brand leading with unfamiliar product, the equilibrium price charged by the single-product firm is $\phi \approx 0.994, p^{U*} \approx 0.896$, and the industry profit is approximately $0.00427/\bar{c}$. Therefore, the targeted recommendation of the two-product brand leading with consumers' familiar product is the optimal strategy by the platform. □

Proof of Proposition 4. If the two-product firm creates two brands with distinct products, the optimal prices for both products are given by (6). We have $\frac{\partial \hat{p}}{\partial v}=\frac{2\hat{p}}{2(1-v)+\hat{p}+3\hat{p}^2}>0$. As the search order requires $\hat{p}>\sqrt{2v-1}$, we have $2v-3\hat{p}<2v-3\sqrt{2v-1}<0$. Then we can get the conditions $v<2-\sqrt{2}$ and $\hat{p}\in(2v/3,\sqrt{2}-1)$ for the equilibrium to hold. The maximized profit for the two-product firm is $\frac{\hat{p}((1-\hat{p})^2+2\hat{p}^2)}{4\bar{c}}$.

First we compare with the separating equilibrium under full recommendation. As the single-product firm earns the same profit as in separating equilibrium without ban, the platform earns a higher profit than the separating equilibrium case without ban if the two-product firm also earns a higher profit, i.e.

$$\frac{\hat{p}((1-\hat{p})^2+2\hat{p}^2)}{4\bar{c}}>\frac{v(1-v)^2}{2\bar{c}}.$$

As the left hand side is increasing in $\hat{p}$ (and hence increasing in v) whereas the right hand side is strictly decreasing in v, it is sufficient to show that the inequality holds when $v=1/2$. When $v=1/2$, $\hat{p}\approx 0.376086$, the left hand side equals $0.0632/\bar{c}$, and the right hand side equals $0.0625/\bar{c}$, which is smaller. By induction, the two-product firm earns a higher profit than creating a single brand. Therefore, it is optimal for the two-product firm to create such two distinct brands when the platform presents all available brands to consumers.

Next we compare with the targeted recommendation of unfamiliar product brand without ban. With the ban, the industry profit is

$$\gamma\frac{{p^*}^2(1-p^*)^2}{4\bar{c}}+(1-\gamma)\frac{\hat{p}((1-\hat{p})^2+2\hat{p}^2)}{4\bar{c}}.$$

The first order derivative with respect to γ is constant at

$$\frac{{p^*}^2(1-p^*)^2}{4\bar{c}}-\frac{\hat{p}((1-\hat{p})^2+2\hat{p}^2)}{4\bar{c}}<0.$$

Under the targeted recommendation of unfamiliar product, the industry profit is

$$\frac{1}{\bar{c}}\left(\frac{\gamma}{2-\gamma}\frac{(1-p^U)^2}{2}+\frac{2-2\gamma}{2-\gamma}\frac{(1-v)^2}{2}\right)\left(\gamma\frac{{p^U}^2}{2}+(1-\gamma)v\right).$$

We've already shown in the previous proofs that the first order derivative with respect to γ is strictly decreasing. Taking $\gamma \to 1$, we have $p^U \to p^*$, and the first order derivative of the difference between industry profit under targeted recommendation of unfamiliar product and industry profit under ban is

$$> \left(-\frac{v(1-p^U)^2}{2\bar{c}} + \frac{\hat{p}((1-\hat{p})^2+2\hat{p}^2)}{4\bar{c}}\right) + \left(\frac{p^{U2}}{2\bar{c}}\frac{1-2v}{(2p^U-1)^2} + p^U\frac{(1-p^U)^2}{2\bar{c}}\right)\frac{(1-p^U)^2-(1-v)^2}{(1-p^U)+\frac{1-2v}{(2p^U-1)^2}}$$

For the first part, $-\frac{v(1-p^U)^2}{2}$ is increasing in v with infimum at -0.0625, and $\frac{\hat{p}((1-\hat{p})^2+2\hat{p}^2)}{4\bar{c}}$ is strictly increasing in v with infimum at 0.0632, so the first part is positive. As we have shown in $\partial p^U/\partial\gamma$ that

$$(1-p^U) + \frac{1-2v}{(2p^U-1)^2} < 0,$$

we have

$$\frac{p^{U2}}{2\bar{c}}\frac{1-2v}{(2p^U-1)^2} + p^U\frac{(1-p^U)^2}{2\bar{c}} < -\frac{p^{U2}}{2\bar{c}}(1-p^U) + p^U\frac{(1-p^U)^2}{2\bar{c}} = p^U\frac{(1-p^U)(1-2p^U)}{2\bar{c}} < 0.$$

Therefore, the second part is also positive, so the difference is always increasing in γ. As the difference is negative at $\gamma = 0$ and 0 at $\gamma = 1$, the difference is negative for $\gamma \in (0,1)$. As the targeted recommendation of unfamiliar product brand with optimal mixing ϕ^* yields a even lower industry profit, the platform would sustain such an equilibrium instead of targeted recommendation.

Lastly, we compare the profit of the two-product firm in these two cases. The difference between the profit under targeted recommendation without ban and the profit under ban is

$$\frac{1}{\bar{c}}\left(\frac{\gamma}{2-\gamma}\frac{(1-p^U)^2}{2} + \frac{2-2\gamma}{2-\gamma}\frac{(1-v)^2}{2}\right)v - \frac{\hat{p}((1-\hat{p})^2+2\hat{p}^2)}{4\bar{c}} < \frac{(1-p_1^*)^2}{2\bar{c}}v - \frac{\hat{p}((1-\hat{p})^2+2\hat{p}^2)}{4\bar{c}}.$$

As the first profit is decreasing in v and the second profit is increasing in v, we have the difference is smaller than the value at $v = 1/2$, which is

$$\frac{0.0625}{\bar{c}} - \frac{0.0632}{\bar{c}} < 0.$$

Therefore, the two-product firm earns higher profit under ban than under the targeted recommendation without ban. □

Proof of Proposition 5. We consider the three cases of selective recommendation here. **(1)** For the first case, the two-product firm would set prices to be v for both products. Denote the price set by the single-product firm by p'. The expected consumer surplus of visiting her familiar product brand is $\frac{\gamma}{2-\gamma}(v-p')$, and the expected single-product firm profit of charging price p is $\frac{1}{2}\frac{\gamma(v-p')}{(2-\gamma)\bar{c}}p +$

$\frac{1}{2}\frac{(1-p')^2}{2\bar{c}}p(1-p)$, so the optimal price is given by

$$p' = \frac{1}{2} + \frac{\gamma}{2-\gamma}\frac{v-p'}{(1-p')^2} \in (\frac{1}{2}, p^*).$$

The maximized profit of the two-product firm is $\frac{\gamma}{2-\gamma}\frac{v-p'}{\bar{c}}v$. If the two-product firm deviates to just create one brand with both products and randomly picks the leading product with equal probability, it will pool with the single-product firm offering the same leading product brand. The deviating profit is

$$\frac{1}{2}\frac{\gamma(v-p')}{(2-\gamma)\bar{c}}v + \frac{1}{2}\frac{(1-p')^2}{2\bar{c}}v > \frac{\gamma}{2-\gamma}\frac{v-p'}{\bar{c}}v.$$

Therefore, this cannot be an equilibrium.

(2) For the second case, the two-product firm would set prices to be 1/2 for both products. Denote the price set by the single-product firm by p''. The expected single-product firm profit of charging price p is

$$\frac{1}{2}\frac{v-p''}{\bar{c}}p + \frac{1}{2\bar{c}}\left(\frac{\gamma}{2-\gamma}\frac{(1-p'')^2}{2} + \frac{2-2\gamma}{2-\gamma}\frac{1}{8}\right)p(1-p),$$

so the optimal price is given by

$$p'' = \frac{1}{2} + \frac{v-p''}{\frac{\gamma}{2-\gamma}(1-p'')^2 + \frac{2-2\gamma}{2-\gamma}\frac{1}{4}} \in (p^*, v).$$

The maximized profit of the two-product firm is $\left(\frac{\gamma}{2-\gamma}\frac{(1-p'')^2}{2} + \frac{2-2\gamma}{2-\gamma}\frac{1}{8}\right)\frac{1}{4\bar{c}}$. If the firm deviates to just create one brand with both products and randomly picks the leading product with equal probability, the deviating profit is

$$\frac{1}{2}\frac{v-p''}{\bar{c}}v + \frac{1}{2\bar{c}}\left(\frac{\gamma}{2-\gamma}\frac{(1-p'')^2}{2} + \frac{2-2\gamma}{2-\gamma}\frac{1}{8}\right)v > \left(\frac{\gamma}{2-\gamma}\frac{(1-p'')^2}{2} + \frac{2-2\gamma}{2-\gamma}\frac{1}{8}\right)\frac{1}{4\bar{c}}.$$

Therefore, this cannot be an equilibrium.

(3) For the third case, the platform randomly shows consumers one brand when there are two distinct brands. All firms would just set product price to be p^*. The two-product firm would earn the same profit as a single-product firm, which has been shown to be lower than if it includes both products in one brand. Therefore, this cannot be an equilibrium.

(4) When $v > 2-\sqrt{2}$, there is no equilibrium where the two-product firm creates two distinct brands. If it just creates a single brand, we've already shown in Lemma 3 that the symmetric single-brand equilibrium is the unique equilibrium. □

Online Appendix

Proof of Proposition 6. **(1)** For the first platform strategy, if each consumer is presented a single brand offering only their unfamiliar product, the price will be $1/2$ with consumer surplus $1/8$ and industry profit $1/4$. This is only feasible when $c \leq 1/8$.

(2) For the second platform strategy, if each consumer is presented a single brand offering both products, it could be targeted recommendation with leading product being their familiar (or unfamiliar) product or non-targeted recommendation with a random leading product. Under targeted recommendation, without loss of generality, assume the leading product is consumers' familiar product. The equilibrium prices will be $(v, (1+v)/2)$ for leading and non-leading products, with consumer surplus $(1-v)^2/8$ and industry profit $(1+v)^2/4$. This is only feasible when $c \leq (1-v)^2/8$. With non-targeted recommendation, the equilibrium prices will be (v, v) with consumer surplus $(1-v)^2/2$ and industry profit v. This is only feasible when $c \leq (1-v)^2/2$.

(3) For the third platform strategy, if each consumer is presented a list of brands offering only their unfamiliar product and let the price of the unfamiliar product be $p^{\infty,U}$, then for a consumer with search cost c, the optimal strategy is to buy immediately when the utility $u \geq d_1$ for some cutoff d_1 and continue search otherwise. The cutoff d_1 is determined by

$$\int_{d_1}^{1} (u - d_1) du = c,$$

so we have

$$d_1 = 1 - \sqrt{2c}.$$

The single-product firm would choose price

$$p^{\infty,U} = \arg\max_p p(1 - d_1 + p^{\infty,U} - p) = \sqrt{2c},$$

with maximized profit conditional on visit to be $2c$. For a consumer with search cost c to start search, it has to be

$$c \leq \int_{p^{\infty,U}}^{1} (u - p^{\infty,U}) du = \frac{(1 - p^{\infty,U})^2}{2} \iff c \leq 1/8.$$

For the platform, the industry profit is $\sqrt{2c}$. Comparing the above three strategies, when $c \leq (1-v)^2/8$, the second platform strategy with targeted recommendation is optimal; when $(1-v)^2/8 < c \leq (1-v)^2/2$, the second platform strategy with non-targeted recommendation is optimal; when $(1-v)^2/2 < c \leq 1/32$, the first platform strategy is optimal; when $\max\{(1-v)^2/2, 1/32\} \leq c < 1/8$, the third platform strategy is optimal.

(4) For the fourth platform strategy, first consider the targeted recommendation that each consumer is presented a list of two-product brands with their familiar product as the leading product. The two-product firms may charge different prices for the two products. Without loss of generality, consider type-L consumers and product 1 as the leading product. In the symmetric equilibrium, all firms charge the same price p_1^∞ for product 1 and the same price p_2^∞ for product 2. Consumers have three choices after searching each brand: buy product 1, buy product 2, and continue search. We first make pairwise comparisons. For a type-L consumer with search cost c, comparing buying unfamiliar product 2 versus continuing search, the optimal strategy is to buy immediately when the utility $u \geq d_2 = d_1$ and continue search otherwise. The two-product firm can, however, charge a relative low price for product 1 to keep this consumer. Comparing buying product 1 versus continuing search, a type-L consumer buys product 1 if

$$\int_{v-p_1^\infty+p_2^\infty}^{1} ((u-p_2^\infty)-(v-p_1^\infty))\, du \leq c,$$
$$\iff p_1^\infty \leq d_3 \equiv p_2^\infty - (1-v) + \sqrt{2c}.$$

Lastly, comparing buying product 1 versus buying product 2, a type-L consumer buys product 1 if $p_1^\infty < v - u + p_2^\infty$ and buys product 2 otherwise. As consumers would not buy product 2 if $u < d_2$, the relevant cutoff for price of product 1 is $d_4 \equiv v - d_2 + p_2^\infty$. As $d_2 = 1 - \sqrt{2c}$, we have $d_3 = d_4$. To maximize profit, the two-product firm would set product 1 price to be $p_1^\infty = d_3$ so that type-L consumers would buy either product 1 or product 2 from the first brand she visits at the highest possible price. (Otherwise, if the product 1 price is even higher, type-L consumers would never consider product 1; if the product 1 price is lower, the two-product firm would find it optimal to increase price.) The two-product firm would choose price p_2 such that

$$p_2^\infty = \arg\max_{p_2} p_2(1-v+p_1^\infty-p_2) + p_1^\infty(v-p_1^\infty+p_2) = p_1^\infty + \frac{1-v}{2}.$$

This contradicts with the condition $p_1^\infty = d_3$. Therefore, this cannot be an equilibrium.

A non-targeted recommendation for the fourth platform strategy is to present each consumer the list of two-product brands with the same leading product, but equally likely to be product 1 or product 2. The firm would charge prices symmetrically for the two products expecting equal demand from both types of consumers. All firms charge prices $(p^{\infty,H}, p^{\infty,L})$ and $(p^{\infty,L}, p^{\infty,H})$ with equal probability for the two products in equilibrium where $p^{\infty,H} \geq p^{\infty,L}$. Consumers have the optimal strategy to buy immediately when the net utility from purchase exceeds some cutoff d_5 and continue search otherwise. The cutoff should satisfy $d_5 \geq v - p^{\infty,H}$, otherwise all consumers would find it better to buy her familiar product than continuing search. There are two cases: (1) $d_5 \leq v - p^{\infty,L}$ that consumers would buy her familiar product if it has price $p^{\infty,L}$; (2) $d_5 > v - p^{\infty,L}$ that consumers would never buy her familiar product. For the first case, the cutoff should satisfy

$$\frac{1}{2}\int_{d_5+p^{\infty,L}}^{1} (u-p^{\infty,L}-d_5)du$$
$$+\frac{1}{2}\left(\int_{v-p^{\infty,L}+p^{\infty,H}}^{1} (u-p^{\infty,H}-d_5)du + (v-p^{\infty,L}+p^{\infty,H})(v-p^{\infty,L}-d_5)\right) = c,$$

which can be simplified as

$$(1-p^{\infty,L}-d_5)^2+(1-p^{\infty,H}-d_5)^2-(v-p^{\infty,L}-d_5)^2+2(v-p^{\infty,L}+p^{\infty,H})(v-p^{\infty,L}-d_5)=4c \quad (1)$$

Say if the two-product firm charges $p_1 \le v-d_5$ and $p_2 > v-d_5$, it will maximize

$$\frac{1}{2}\left(p_1(v-p_1+p_2)+p_2(1-v+p_1-p_2)\right)+\frac{1}{2}p_1(1-d_5-p_1)$$

The optimal prices given the constraints are

$$p^{\infty,L}=v-d_5,\ p^{\infty,H}=\frac{1+v}{2}-d_5.$$

However, it can be shown that the cutoff condition (1) does not hold in general with these $p^{\infty,L}, p^{\infty,H}$. For the second case, the firm would simply charge $p^{\infty,H}=p^{\infty,L}=\sqrt{2c}$, and is feasible only when $c \le (1-v)^2/2$. The familiar product is never purchased, so the equilibrium outcome is the same as targeted recommendation of a list of brands with only unfamiliar product. □

Proof of Lemma 4. For the two-product firm, given a fraction α of consumers being type-L, if the firm sets both prices to be v, then consumers buy one of the two products for sure and the firm earns conditional profit v. If the firm sets both prices to be $1/2$, they can only sell each consumer their unfamiliar product when the valuation is high. The expected conditional profit is $1/4$. Comparing these two uniform pricing cases, setting (v, v) is better when $v > 1/4$, and setting $(1/2, 1/2)$ is better when $v < 1/4$. However, the two-product firm can also charge different prices for these two products. Similar to the main model, the optimal asymmetric prices would be $(v,(1+v)/2)$ or $((1+v)/2, v)$ conditional on consumer visit. If the firm sets the price of product 1 to be v and the other to be $(1+v)/2$, the expected conditional profit is

$$\alpha\left(\frac{1-v}{2}\cdot\frac{1+v}{2}+\frac{1+v}{2}v\right)+(1-\alpha)(1-v)v=v(1-v)+\alpha\frac{4v^2+(1-v)^2}{4}.$$

Similarly we have the expected conditional profit by setting the price of product 1 to be $(1+v)/2$ and price of product 2 to be v. If $\alpha \ge 1/2$, setting the prices to be $(v,(1+v)/2)$ would yield higher profit, and vice versa. Finally, we compare all four pricing strategies to find the optimal strategies of the two-product firm conditional on α given by

$$\begin{cases} (v,v) & \text{if } v \ge \frac{1}{3} \text{ and } \frac{(1-v)^2}{4v^2+(1-v)^2} \le \alpha \le \frac{4v^2}{4v^2+(1-v)^2}, \\ (\frac{1}{2},\frac{1}{2}) & \text{if } v \le 1-\frac{\sqrt{6}}{3} \text{ and } \frac{v(2+v)}{4v^2+(1-v)^2} \le \alpha \le \frac{(1-2v)^2}{4v^2+(1-v)^2}, \\ (v,\frac{1+v}{2}) & \text{otherwise and } \alpha \ge \frac{1}{2}, \\ (\frac{1+v}{2},v) & \text{otherwise and } \alpha \le \frac{1}{2}. \end{cases}$$

To analyze the equilibrium, we need to solve consumers' strategies. For a type-L consumer, the expected surplus of visiting a brand with only her familiar product is 0 at expected price v and

$v-1/2<0$ at expected price $1/2$. Therefore, a consumer would never visit a brand if she expects it only offers her familiar product. The expected surplus of visiting a brand with only unfamiliar product is $(1-v)^2/2$ at expected price v and $1/8$ at expected price $1/2$. For a type-L consumer, the expected surplus of visiting a two-product brand with prices at (v,v) is $(1-v)^2/2$; the expected surplus of visiting a two-product brand with prices at $(1/2,1/2)$ is $1/8$; the expected surplus of visiting a two-product brand with prices at $((1+v)/2,v)$ is $(1-v)^2/2$; the expected surplus of visiting a two-product brand with prices at $(v,(1+v)/2)$ is $(1-v)^2/8$; the expected surplus of visiting a two-product brand with prices at $((1+v)/2,v)$ and $(v,(1+v)/2)$ with equal probability is $5(1-v)^2/16$.

Now we analyze the two-brand equilibrium under full recommendation. If the two-product firm creates two brands with distinct leading products, similar to the $v>1/2$ case, the firm will symmetrically charge the prices for two products charge the same prices for both brands it creates, attracting equal proportion of consumers to each brand. The difference is that the pricing strategy is more complicated. As shown above, the two-product firm will choose prices $(1/2,1/2)$ when $v\leq 1-\sqrt{6}/3$, choose $((1+v)/2,v)$ and $(v,(1+v)/2)$ with equal probability when $1-\sqrt{6}/3<v\leq 1/3$, and choose (v,v) when $1/3<v<1/2$. If the two-product firm chooses $((1+v)/2,v)$ and $(v,(1+v)/2)$ with unequal probability, consumers would visit each brand with unequal proportion, and the firm would switch the probabilities to maximize profit, so it cannot be an equilibrium. □

Proof of Proposition 7. In the two-brand equilibrium under full recommendation, the equilibrium industry profit is

$$R_1=\begin{cases}\frac{2-\gamma}{64\bar{c}} & \text{if } 0<v\leq 1-\frac{\sqrt{6}}{3},\\ \frac{\gamma}{64\bar{c}}+(1-\gamma)\frac{5(1-v)^2}{128\bar{c}}\left(4v(1-v)+(1+v)^2\right) & \text{if } 1-\frac{\sqrt{6}}{3}<v\leq\frac{1}{3},\\ \frac{\gamma}{64\bar{c}}+(1-\gamma)\frac{v(1-v)^2}{2\bar{c}} & \text{if } \frac{1}{3}<v<\frac{1}{2}.\end{cases}$$

Consider the single-brand equilibrium where the two-product firm just creates one brand, equally likely to be leading-1 and leading-2. If the two-product firm charges (v,v), the expected consumer surplus of type-L consumers visiting a leading-1 brand is $(1-\gamma)(1-v)^2/2$ and the expected consumer surplus of type-R consumers visiting a leading-1 brand is $\gamma/8+(1-\gamma)(1-v)^2/2<(1-v)^2/2$, so the average consumer traffic is lower than under the two-brand equilibrium. If the two-product firm charges $(1/2,1/2)$, the expected consumer surplus of type-L consumers visiting a leading-1 brand is $(1-\gamma)/8$ and the expected consumer surplus of type-R consumers visiting a leading-1 brand is $1/8$, so the average consumer traffic is also lower than under the two-brand equilibrium. Similarly for choosing $((1+v)/2,v)$ and $(v,(1+v)/2)$ with equal probability. Therefore, the two-product firm has no incentive to pool with the single-product firm as it attracts more consumer traffic on average. In comparison with the single-brand equilibrium, the single-product firm is worse off in two-brand equilibrium with fewer consumer visits.

Lastly, we consider the targeted recommendation when the single-product firm charges $1/2$ for its sole product, the two-product firm charges $((1+v)/2, v)$ for leading-1 brand and $(v,(1+v)/2)$ for leading-2 brand, and the platform targetedly recommend the two-product brand leading with consumers' unfamiliar product. For instance, the expected consumer surplus of type-L consumers visiting a leading-1 brand is 0, while the expected consumer surplus of type-R consumers visiting a leading-1 brand is

$$\frac{\gamma}{2-\gamma}\frac{1}{8}+\frac{2-2\gamma}{2-\gamma}\frac{(1-v)^2}{8}.$$

The industry profit is

$$R_2=\left(\frac{\gamma}{2-\gamma}\frac{1}{8\bar{c}}+\frac{2-2\gamma}{2-\gamma}\frac{(1-v)^2}{8\bar{c}}\right)\left(\frac{\gamma}{8}+(1-\gamma)\frac{(1+v)^2}{4}\right).$$

The partial derivative with respect to v is

$$\begin{aligned}&-\frac{2-2\gamma}{2-\gamma}\frac{1-v}{4\bar{c}}\left(\frac{\gamma}{8}+(1-\gamma)\frac{(1+v)^2}{4}\right)+\left(\frac{\gamma}{2-\gamma}\frac{1}{8\bar{c}}+\frac{2-2\gamma}{2-\gamma}\frac{(1-v)^2}{8\bar{c}}\right)(1-\gamma)\frac{1+v}{2}\\ =&\frac{1-\gamma}{2-\gamma}\frac{v}{8\bar{c}}\left(\gamma-2(1-\gamma)(1-v^2)\right),\end{aligned}$$

where inside the parenthesis is increasing in v. Therefore, R_2 is always increasing in v, or always decreasing in v, or first decreasing and then increasing in v. When $v=0$, $R_2=(2-\gamma)/(64\bar{c})$.

When $0<v\le 1-\sqrt{6}/3$, we can easily solve the condition for $R_2<R_1$ to be $\gamma<(2-v^2)/(3-v^2)$. When $1-\sqrt{6}/3<v\le 1/3$, similarly it can be shown that $\partial(R_2-R_1)/\partial v$ is always increasing in v, so R_2-R_1 is always increasing in v, or first decreasing and then increasing in v. As $R_2-R_1<0$ when $v=1-\sqrt{6}/3$ and $R_2-R_1<0$ when $v=1/3$, $R_2<R_1$ for any $v\in(1-\sqrt{6}/3,1/3)$. When $1/3<v<1/2$, similarly it can be shown that $\partial(R_2-R_1)/\partial v$ is always increasing in v, and R_2-R_1 is always increasing in v, or always decreasing in v, or first decreasing and then increasing in v. As $R_2-R_1<0$ when $v=1/3$ and $R_2-R_1<0$ when $v=1/2$, $R_2<R_1$ for any $v\in(1/3,1/2)$.

Therefore, the industry profit under targeted recommendation is lower than that under full recommendation in each of the three parameter segments expect for when $0<v\le 1-\sqrt{6}/3$ and $\gamma>(2-v^2)/(3-v^2)$, in which case the platform would choose targeted recommendation. □

Proof of Proposition 8. As shown in our previous analysis, for the two-brand equilibrium with the platform choosing full recommendation, the total expected sales volume is

$$\gamma\frac{p^*(1-p^*)^2}{4\bar{c}}+(1-\gamma)\frac{(1-v)^2}{2\bar{c}}. \tag{2}$$

(1) We prove that the two-brand equilibrium is the unique equilibrium here. If the platform randomly shows one of the brands with equal probability when the two-product firm creates two distinct brands, or if the two-product firm creates just one brand, equally likely to be leading-1 or leading-2, it will be the pooling equilibrium we analyzed. The sales volume of the two-product firm is lower than that under the separating equilibrium if

$$\gamma\frac{(1-p^P)^2}{2}+\gamma(v-p^P)+(1-\gamma)(1-v)^2<(1-v)^2 \Leftrightarrow\ v-p^P+\frac{(1-p^P)^2}{2}<(1-v)^2,$$

which holds as shown before. The sales volume of the single-product firm is also lower if

$$p^P\left(\gamma(1-p^P)^2+(1-\gamma)(1-v)^2\right)<p^*(1-p^*)^2 \Leftarrow\ p^P(1-p^P)^2<p^*(1-p^*)^2,$$

which always holds as $p^P > p^* > \frac{1}{2}$. For the platform, the total trade volume is a weighted average of single-product and two-product firm types. Therefore, we still have the result that the platform would do better by showing all available brands and sustain the separating equilibrium in the privacy environment.

(2) We prove that the platform will choose targeted recommendation. Consider selective recommendation. For the targeted recommendation of familiar product brand, the sales volume of the two-product firm is lower than that under the separating equilibrium if

$$\frac{\gamma}{2-\gamma}(v-p^F)+\frac{2-2\gamma}{2-\gamma}\frac{(1-v)^2}{2}<\frac{(1-v)^2}{2} \Leftrightarrow\ v-p^F<\frac{(1-v)^2}{2},$$

The sales volume of the single-product firm is also lower if

$$p^F(1-p^F)^2<p^*(1-p^*)^2,$$

which always holds as $p^F > p^* > \frac{1}{2}$. Therefore, the total expected sales volume is lower for the platform than full recommendation. Similarly, for the targeted recommendation of unfamiliar product, the total expected sales volume is

$$\frac{1}{\bar{c}}\left(\frac{\gamma}{2-\gamma}\frac{(1-p^U)^2}{2}+\frac{2-2\gamma}{2-\gamma}\frac{(1-v)^2}{2}\right)\left(\gamma\frac{p^U}{2}+(1-\gamma)\right). \tag{3}$$

It is easy to see that the above two expected sales volumes (2) and (3) are the same when $\gamma \in \{0,1\}$. The partial derivative of the total expected sales volume (2) in separating equilibrium with respect to γ is constant at

$$\frac{p_1^*(1-p_1^*)^2}{4\bar{c}}-\frac{(1-v)^2}{2\bar{c}}<0.$$

The first order partial derivative of the total expected sales volume (3) in targeted recommendation with respect to γ is

$$\frac{1}{\bar{c}}\left(\frac{p^U}{2}-1\right)\frac{v-p^U}{2p^U-1}+\frac{1}{\bar{c}}\left(\left(\gamma\frac{p^U}{2}+(1-\gamma)\right)\frac{1-2v}{(2p^U-1)^2}+\frac{\gamma}{2}\frac{v-p^U}{2p^U-1}\right)\frac{\partial p^U}{\partial\gamma}.$$

The second order partial derivative of the targeted industry profit with respect to γ is

$$\frac{1}{\bar{c}}\frac{B_1\frac{\partial p^U}{\partial\gamma}+B_2\left(\frac{\partial p^U}{\partial\gamma}\right)^2}{\gamma(1-p^U)+(2-\gamma)\frac{1-2v}{(2p^U-1)^2}},$$

where the denominator has been shown to be negative, and

$$\begin{aligned}B_1=&\frac{2(2v-1)}{(2p^U-1)^4}\left((1-p^U)(2p^U-1)^2-(v-p^U)(2p^U-1)-(1-p^U)(2v-1)\right)\\<&\frac{2(2v-1)}{(2p^U-1)^3}\left((1-v)p^U(2p^U-1)-(v-p^U)-(1-p^U)\right)<0,\\B_2=&\frac{\gamma(2v-1)}{(2p^U-1)^3}\left(2-\gamma-(2p^U-1)\left(1-\frac{\gamma}{2}p^U\right)\right)+\frac{\gamma^2}{2}\frac{v-p^U}{2p^U-1}>0.\end{aligned}$$

The sign of B_1 (B_2) can be shown by first showing the terms in the big parenthesis is increasing (decreasing) in p^U. Hence $B_1\frac{\partial p^U}{\partial\gamma}+B_2\left(\frac{\partial p^U}{\partial\gamma}\right)^2>0$. Therefore the first order partial derivative of the selective recommendation profit is strictly decreasing in γ while the first order partial derivative of the separating equilibrium profit is constant, so the difference between the targeted industry profit and the separating industry profit must be first increasing and then decreasing in γ. Hence the difference is always above 0 as the differences at the two end points are 0. We show that our results can be extended to this alternative incentive scheme of the platform for both privacy and no privacy environment.

(3) We show that the ban can further benefit the platform. Consider the equilibrium if the platforms bans the identical-menu virtual brands under this alternative incentive scheme. With the ban, the total trade volume is

$$\gamma\frac{p^*(1-p^*)^2}{4\bar{c}}+(1-\gamma)\frac{(1-\hat{p})^2+2\hat{p}^2}{4\bar{c}}.$$

First we compare with the separating equilibrium under full recommendation. As the single-product firm charges the same price as in separating equilibrium without ban, the platform earns a higher profit than the separating equilibrium case without ban if the two-product firm has a higher trade volume, i.e.

$$\frac{(1-\hat{p})^2+2\hat{p}^2}{4\bar{c}}>\frac{(1-v)^2}{2\bar{c}}.$$

As the left hand side is increasing in $\hat{p}$ (and hence increasing in v) whereas the right hand side is strictly decreasing in v, it is sufficient to show that the inequality holds when $v=1/2$. When $v=1/2$, $\hat{p}\approx 0.376086$, the left hand side equals $0.168/\bar{c}$, and the right hand side equals $0.125/\bar{c}$, which is smaller. By induction, the two-product firm has a higher trade volume than creating a single brand. Therefore, the platform earns a higher profit than the separating equilibrium without ban.

Next we compare with the targeted recommendation of unfamiliar product brand without ban.

The first order derivative of total trade volume with respect to γ is constant at

$$\frac{p^*(1-p^*)^2}{4\bar{c}} - \frac{(1-\hat{p})^2 + 2\hat{p}^2}{4\bar{c}} < 0.$$

Under the targeted recommendation without ban, the total trade volume is (3). We've already shown that the first order derivative with respect to γ is strictly decreasing. Taking $\gamma \to 1$, we have $p^U \to p^*$, and the first order derivative of the difference between total trade volume under targeted recommendation without ban and total trade volume under ban is

$$> \left(-\frac{(1-p^U)^2}{2\bar{c}} + \frac{(1-\hat{p})^2 + 2\hat{p}^2}{4\bar{c}}\right) + \left(\frac{p^U}{2\bar{c}}\frac{1-2v}{(2p^U-1)^2} + \frac{(1-p^U)^2}{4\bar{c}}\right)\frac{(1-p^U)^2 - (1-v)^2}{(1-p^U) + \frac{1-2v}{(2p^U-1)^2}}$$

For the first part, $-\frac{v(1-p^U)^2}{2}$ is increasing in v with infimum at -0.125, and $\frac{\hat{p}((1-\hat{p})^2+2\hat{p}^2)}{4\bar{c}}$ is strictly increasing in v with infimum at 0.168, so the first part is positive. As we have shown in $\partial p^U/\partial\gamma$ that $(1-p^U) + \frac{1-2v}{(2p^U-1)^2} < 0$, we have

$$\frac{p^U}{2\bar{c}}\frac{1-2v}{(2p^U-1)^2} + \frac{(1-p^U)^2}{4\bar{c}} < -\frac{p^U}{2\bar{c}}(1-p^U) + \frac{(1-p^U)^2}{4\bar{c}} = \frac{(1-p^U)(1-3p^U)}{4\bar{c}} < 0.$$

Therefore, the second part is also positive, so the difference is always increasing in γ. As the difference is negative at $\gamma = 0$ and 0 at $\gamma = 1$, the difference is negative for $\gamma \in (0, 1)$. Therefore, the platform would sustain such an equilibrium instead of targeted recommendation of brand leading with unfamiliar product. □